

**A particle-level review of**

# **Soil Behavior**

**- *macroscale implications* -**

**J. Carlos Santamarina**  
**KAUST**



# Terzaghi (1883-1963)

"... Coulomb... purposely ignored the fact that **[soils] consist of individual grains**

Coulomb's idea proved very useful as a working hypothesis .... but it developed into an **obstacle against further progress**

[let's start] again from the elementary fact that **[soils] consist of individual grains"**

Terzaghi (1920, ENR)



# Formation

Size → Forces

Shape

Soil Classification

Diagenesis

Shear strength

Stiffness

Pores

Permeability

Mixed fluids



Michelangelo

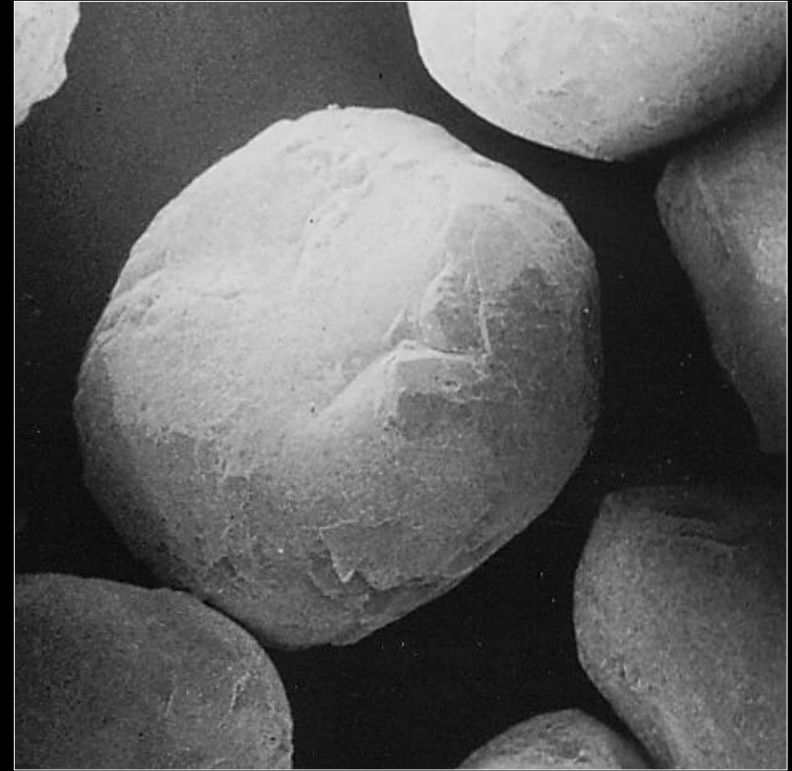
# Mechano-Genesis: Gravels, Sands... <sup>some</sup> Silts

*crushed granite*



100µm

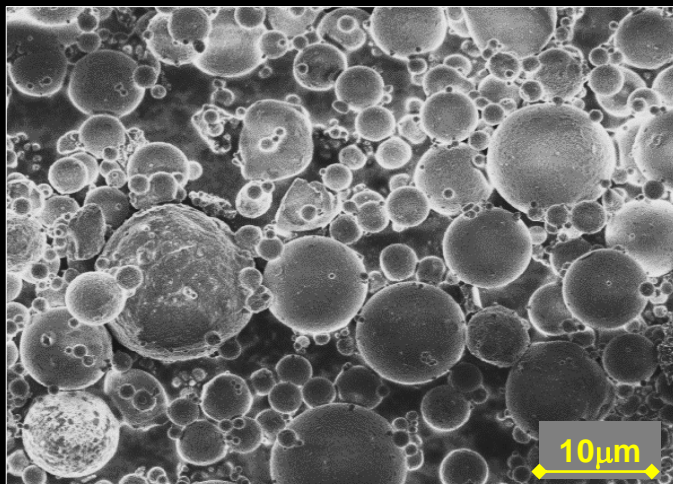
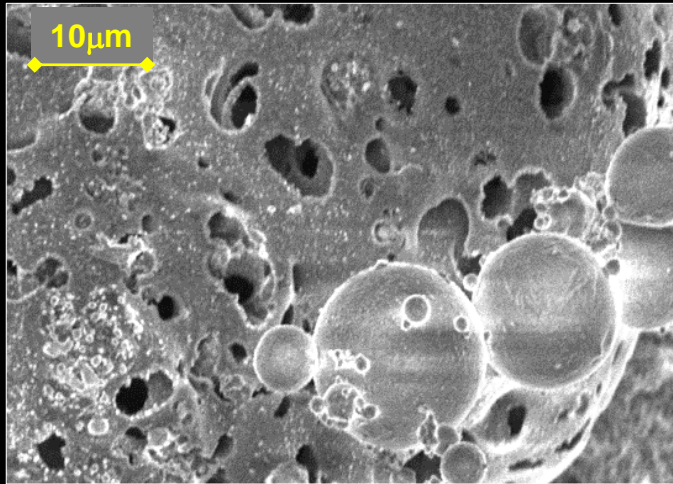
*Ottawa sand*



100µm

# Thermo-Genesis: Ash

## *fly ash*



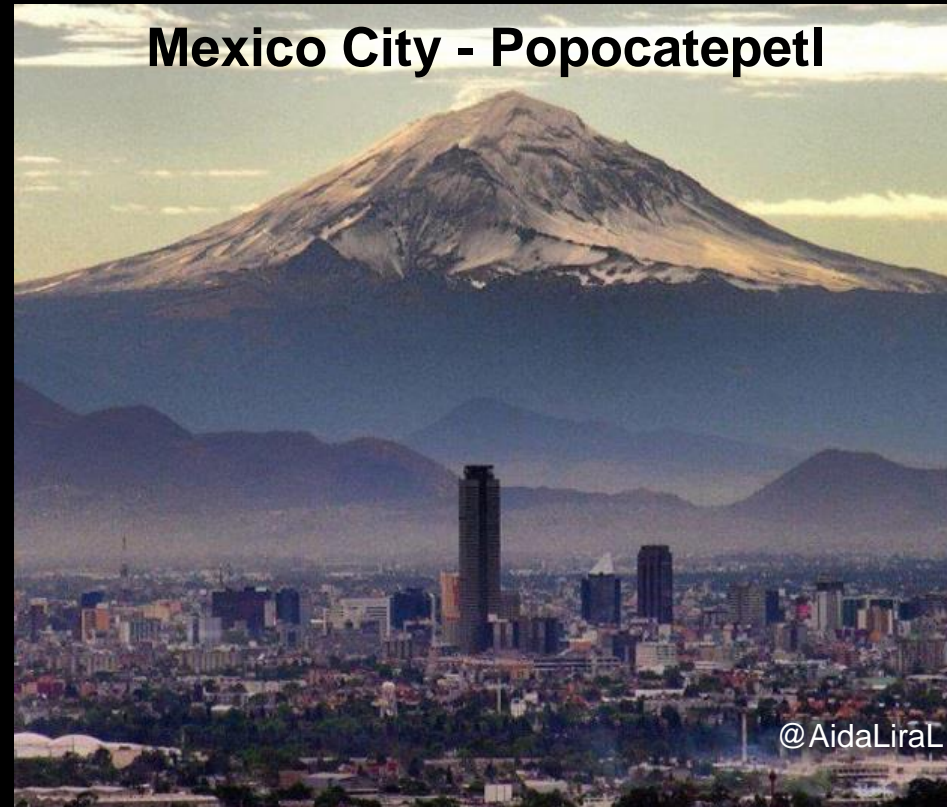
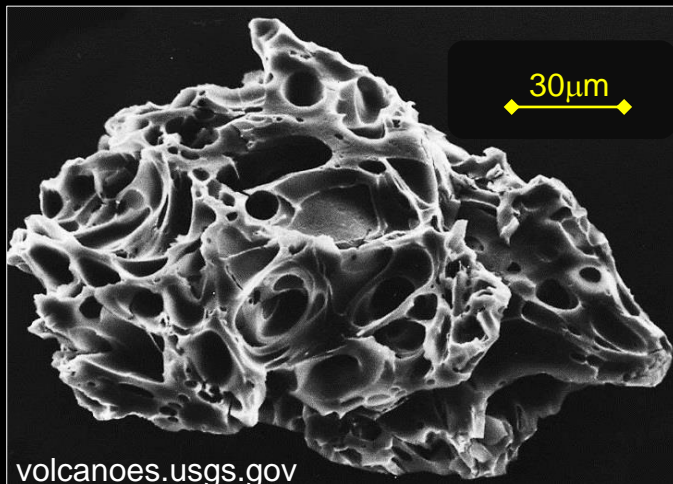
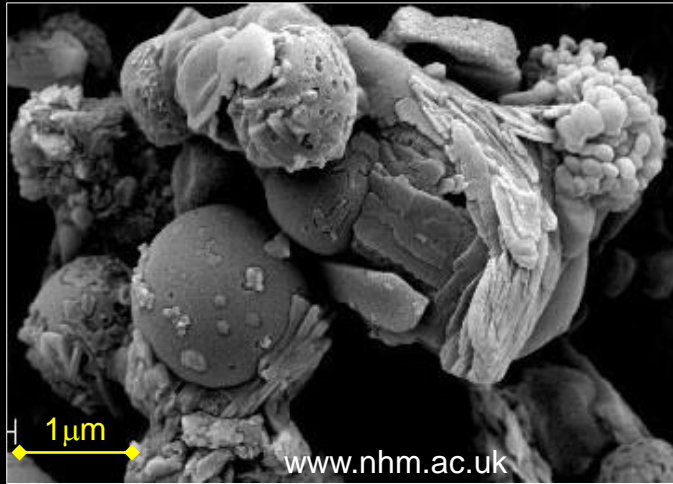
TVA Kingston Fossil Plant – 12/22/2008



(US EPA)

# Thermo-Genesis: Ash

## *volcanic ash*

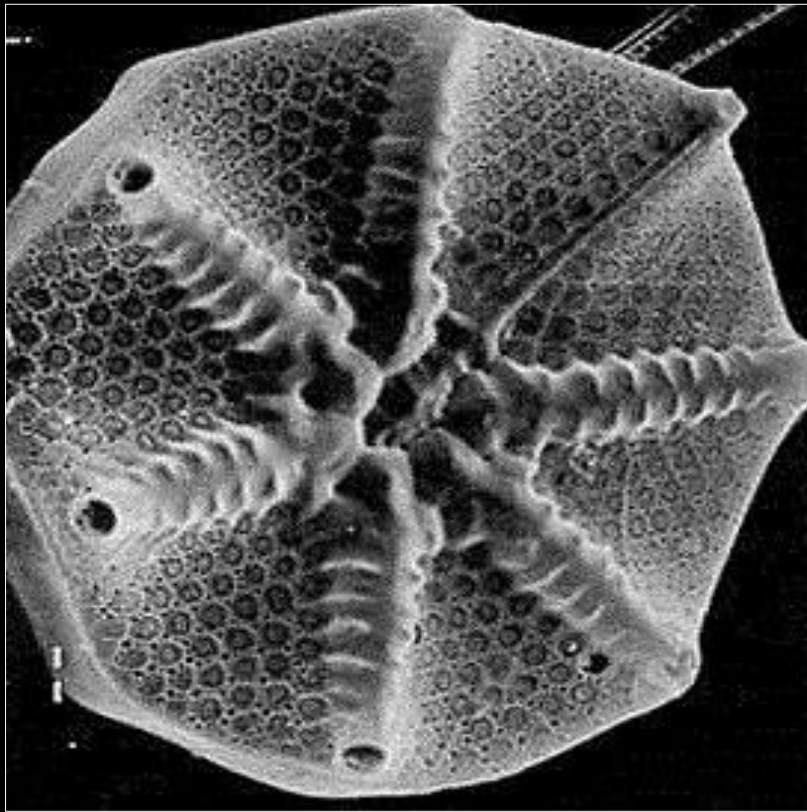


LL > 250

$\phi > 42^\circ$

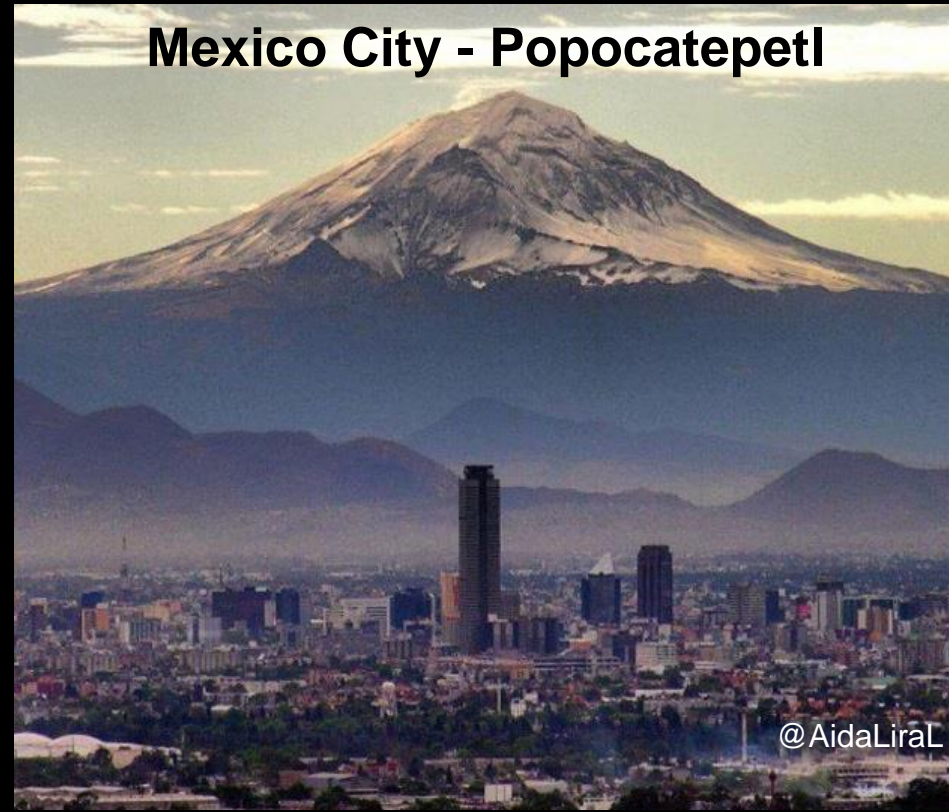


# Bio-Genesis: Diatoms



*The Diatoms –1990*

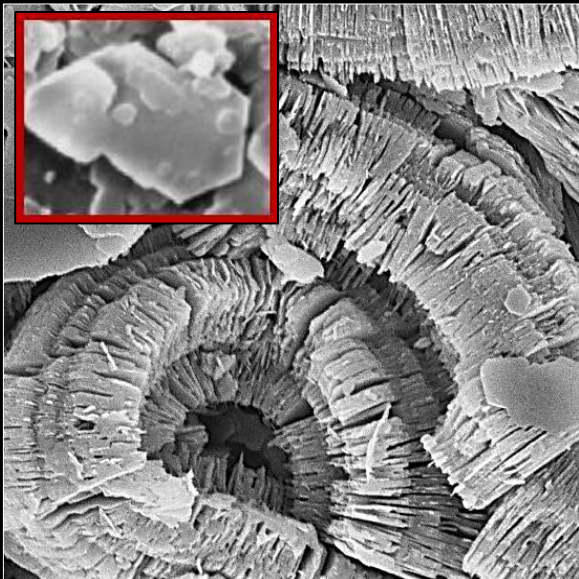
10 $\mu$ m



@AidaLiraL

# Chemo-Genesis: Clays

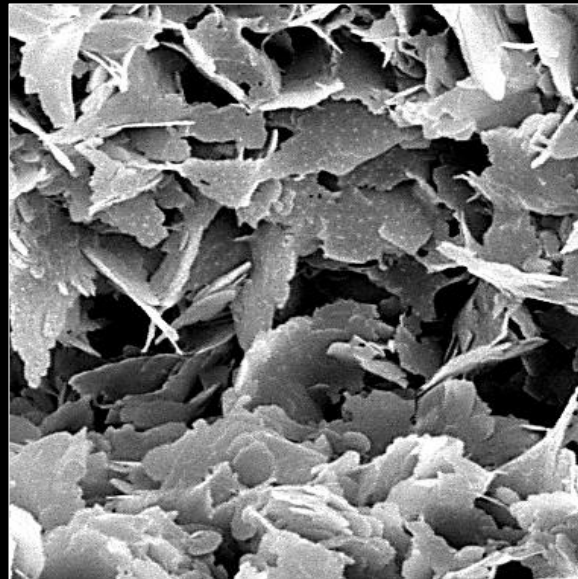
*kaolinite*



2μm

$S_s = 10 \text{ m}^2/\text{g}$

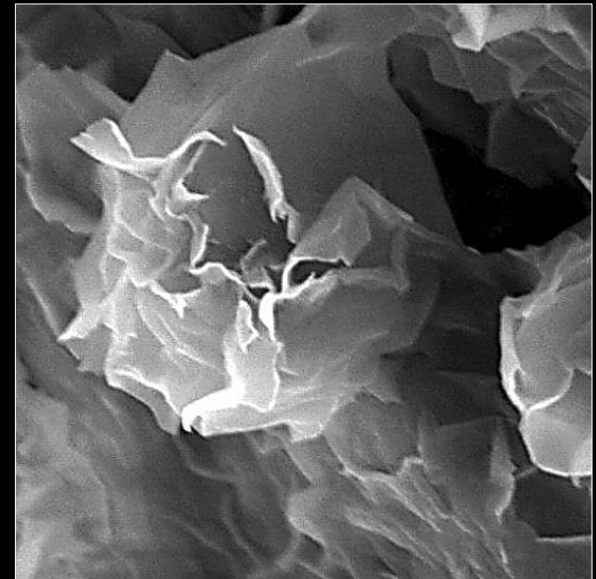
*illite*



2μm

$S_s = 50 \text{ m}^2/\text{g}$

*smectite*



2μm

$S_s = 300 \text{ m}^2/\text{g}$



**Formation**

**Size → Forces**

**Shape**

**Soil Classification**

**Diagenesis**

**Shear strength**

**Stiffness**

**Pores**

**Permeability**

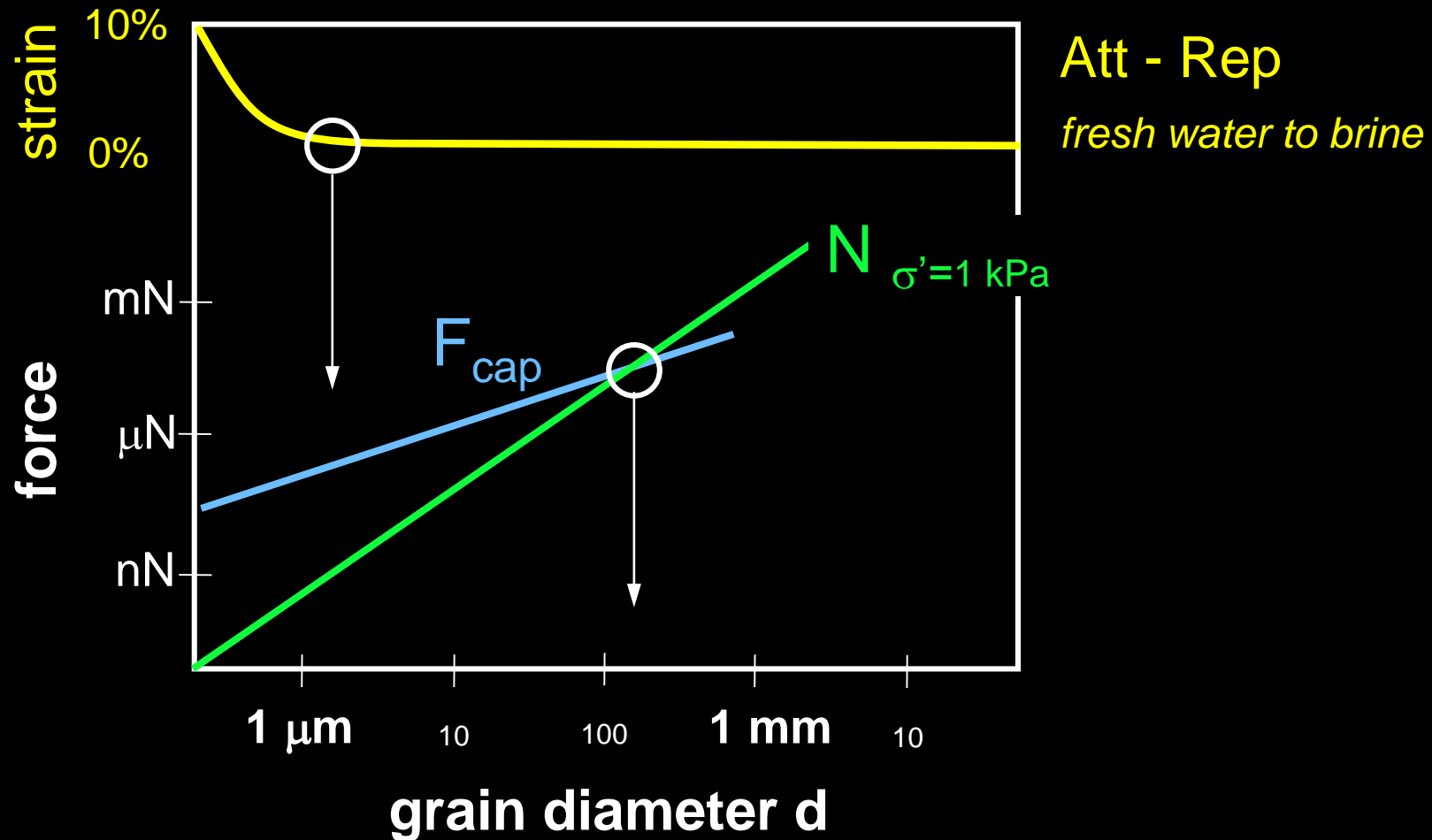
**Mixed fluids**



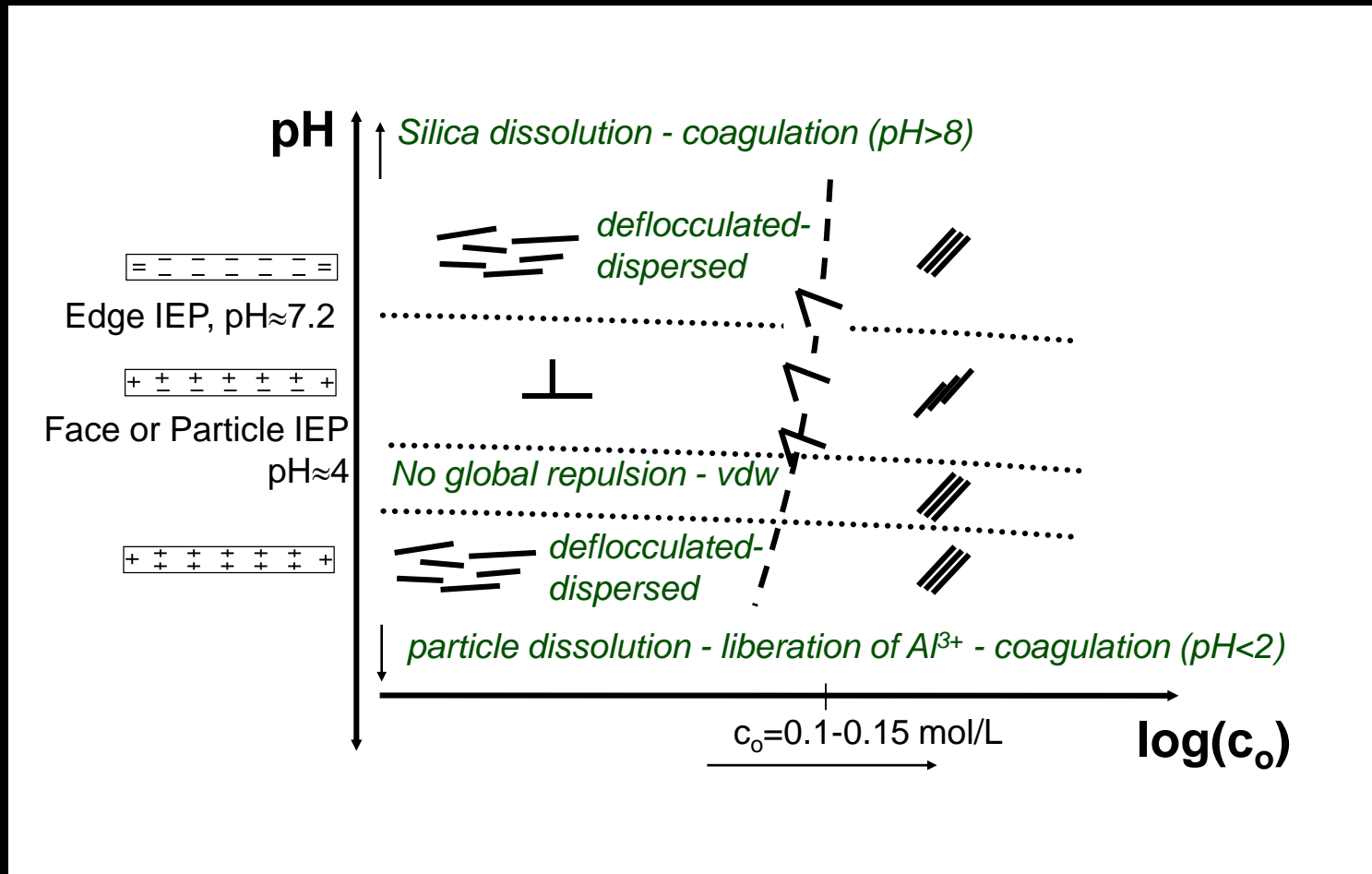
# Particle Forces – Spherical Particles

<b>Skeletal</b>	$\underline{N} = \sigma' d^2$
<b>Weight</b>	$W = (\pi G_s \gamma_w / 6) d^3$
<b>Buoyant</b>	$U = Vol \cdot \gamma_w = (\pi \gamma_w / 6) d^3$
<b>Hydrodynamic</b>	$F_{drag} = 3\pi \mu v d$
<b>Capillary</b>	$F_{cap} = \pi T_s d$
<b>Electrical</b>	
attraction	$Att = \frac{A_h}{24t^2} d$
repulsion	$Rep = 0.0024 \sqrt{c_o} e^{-10^8 t \sqrt{c_o}} d$
<b>Cementation</b>	$T = \pi \sigma_{ten} t d$

# Force Balance: Deformation & Strength



# Fine-Grained Fabric: Fluid dependent



kaolinite

**Formation**

**Size → Forces**

**Shape**

**Soil Classification**

**Diagenesis**

**Shear strength**

**Stiffness**

**Pores**

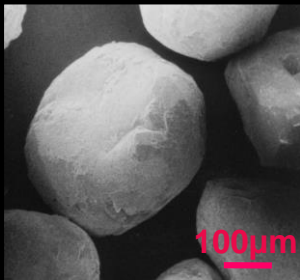
**Permeability**

**Mixed fluids**

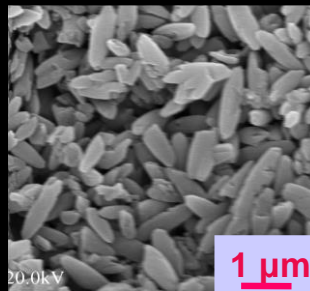


# Particle Shape

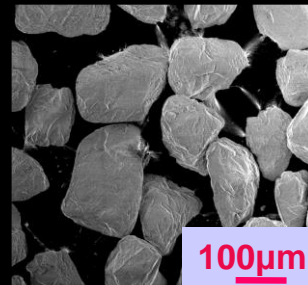
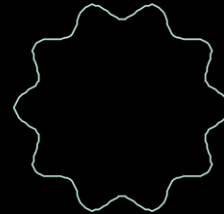
size d



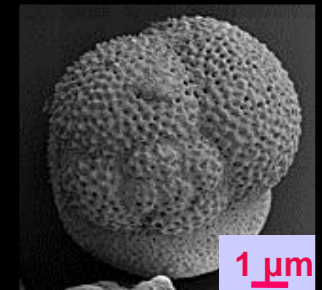
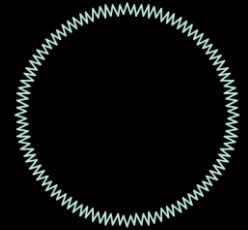
sphericity  
ellipticity..platiness



roundness  
angularity



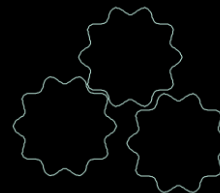
smoothness  
roughness



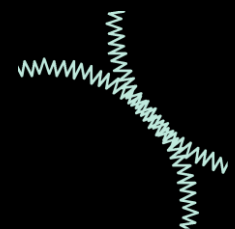
*alignment*



*interlocking*

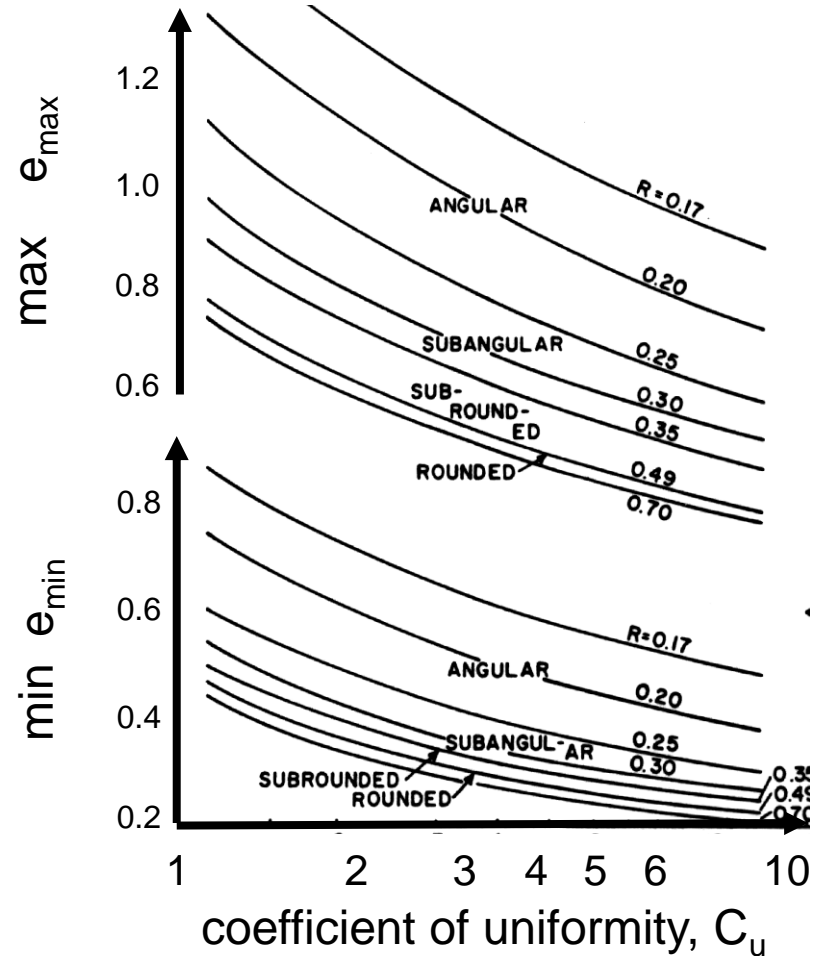
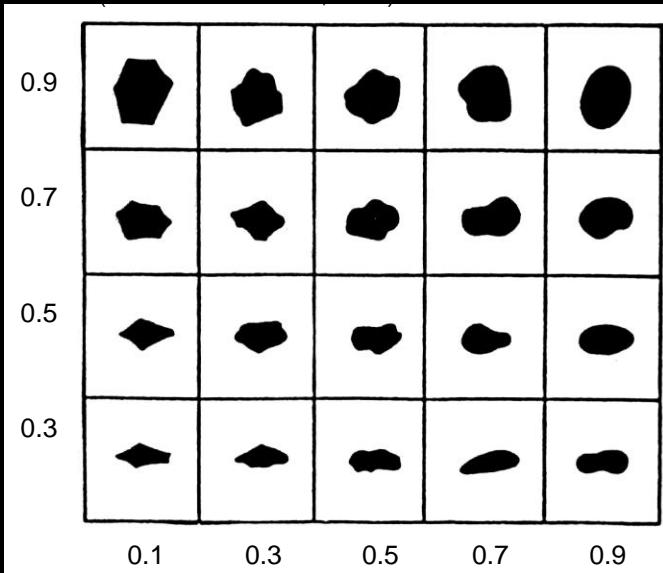
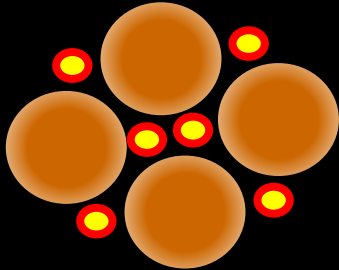


*surface μ*



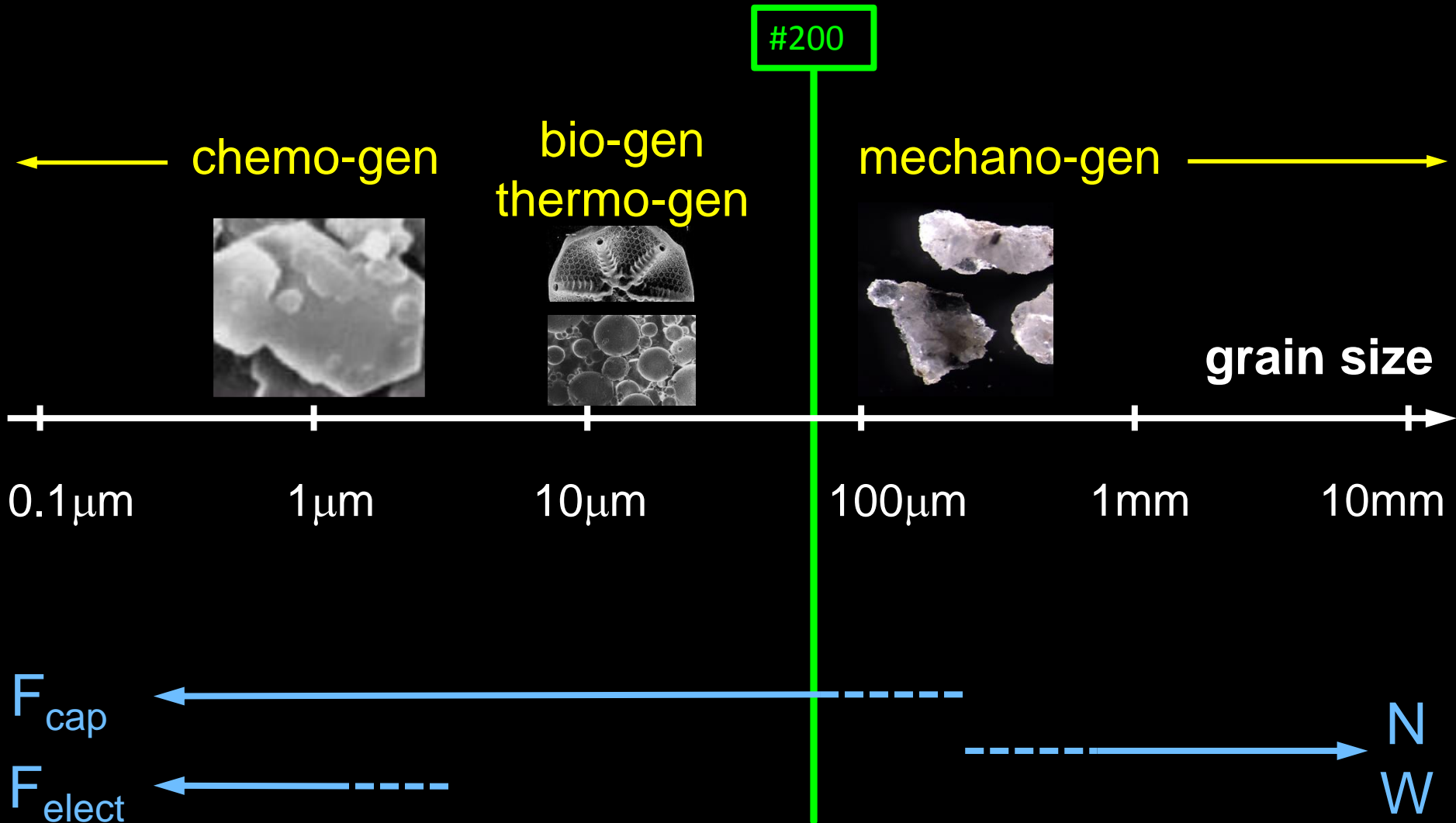
# Coarse Grained: Shape + Relative Size

$C_u$



(Youd, 1973; see also Maeda, 2001)

# Summary: Genesis → Size – Shape – Forces





**Formation**

**Size → Forces**

**Shape**

**Soil Classification**

**Diagenesis**

**Shear strength**

**Stiffness**

**Pores**

**Permeability**

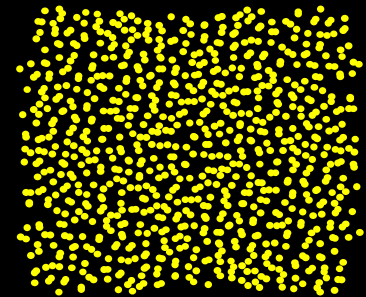
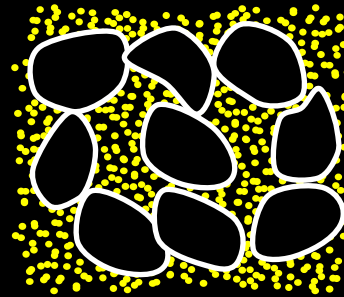
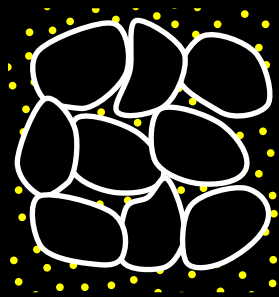
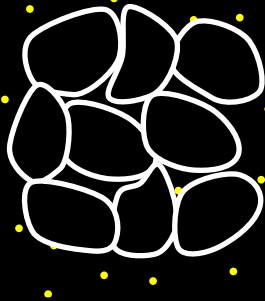
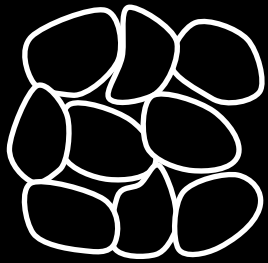
**Mixed fluids**



Casagrande (1902-1981)

# Sand + Gravel

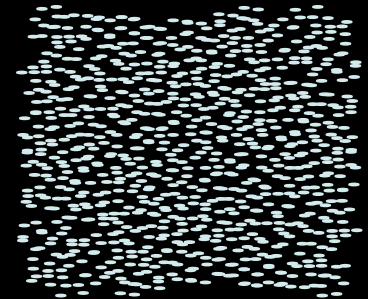
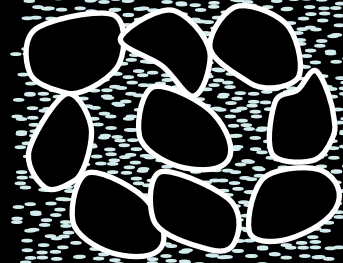
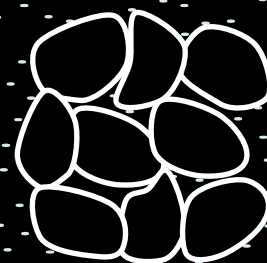
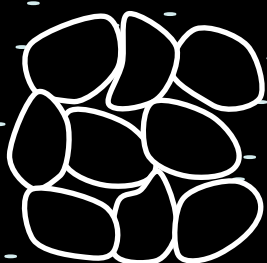
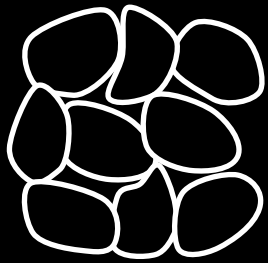
sand fraction:  $\chi = \frac{M_{\text{sand}}}{M_{\text{total}}} = \frac{e_G}{1 + e_G + e_s}$



gravel	$e_G = e_{\min}$	$e_G = e_{\max}$	} $f(C_u, \text{shape})$
sand	$e_S = e_{\max}$	$e_S = e_{\min}$	
sand fraction	$\chi_L = 20-25 \%$	$\chi_H = 35-40 \%$	

# Fine + Coarse

fines fraction:  $\Phi = \frac{M_{\text{fines}}}{M_{\text{total}}} = \frac{e_C}{1 + e_C + e_F}$

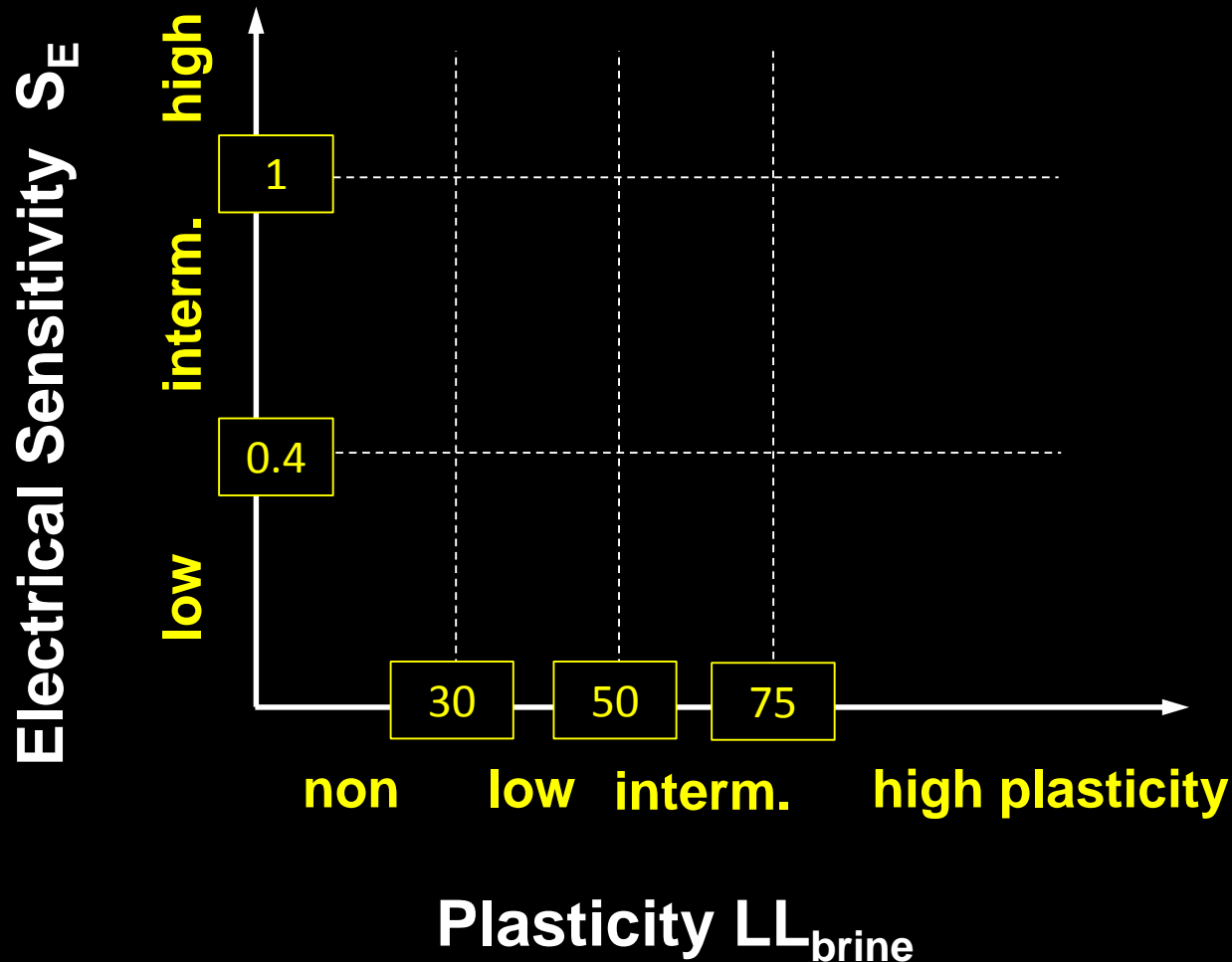


coarse	$e_C = e_{\min}$	$e_C = e_{\max}$	
fines	$e_F = e_{1\text{kPa}}$	$e_F = e_{1\text{MPa}}$	
fines fraction	$\Phi_L = 18\%$	$\Phi_H = 24\%$	kaolin
	$\Phi_L = 8\%$	$\Phi_H = 25\%$	montmorillonite

# Classification: Fines

(Pass #200)

$$S_E = \sqrt{\left(\frac{LL_{DW}}{LL_{brine}} - 1\right)^2 + \left(\frac{LL_{ker}}{LL_{brine}} - 1\right)^2}$$



**Formation**

**Size → Forces**

**Shape**

**Soil Classification**

**Diagenesis**

**Shear strength**

**Stiffness**

**Pores**

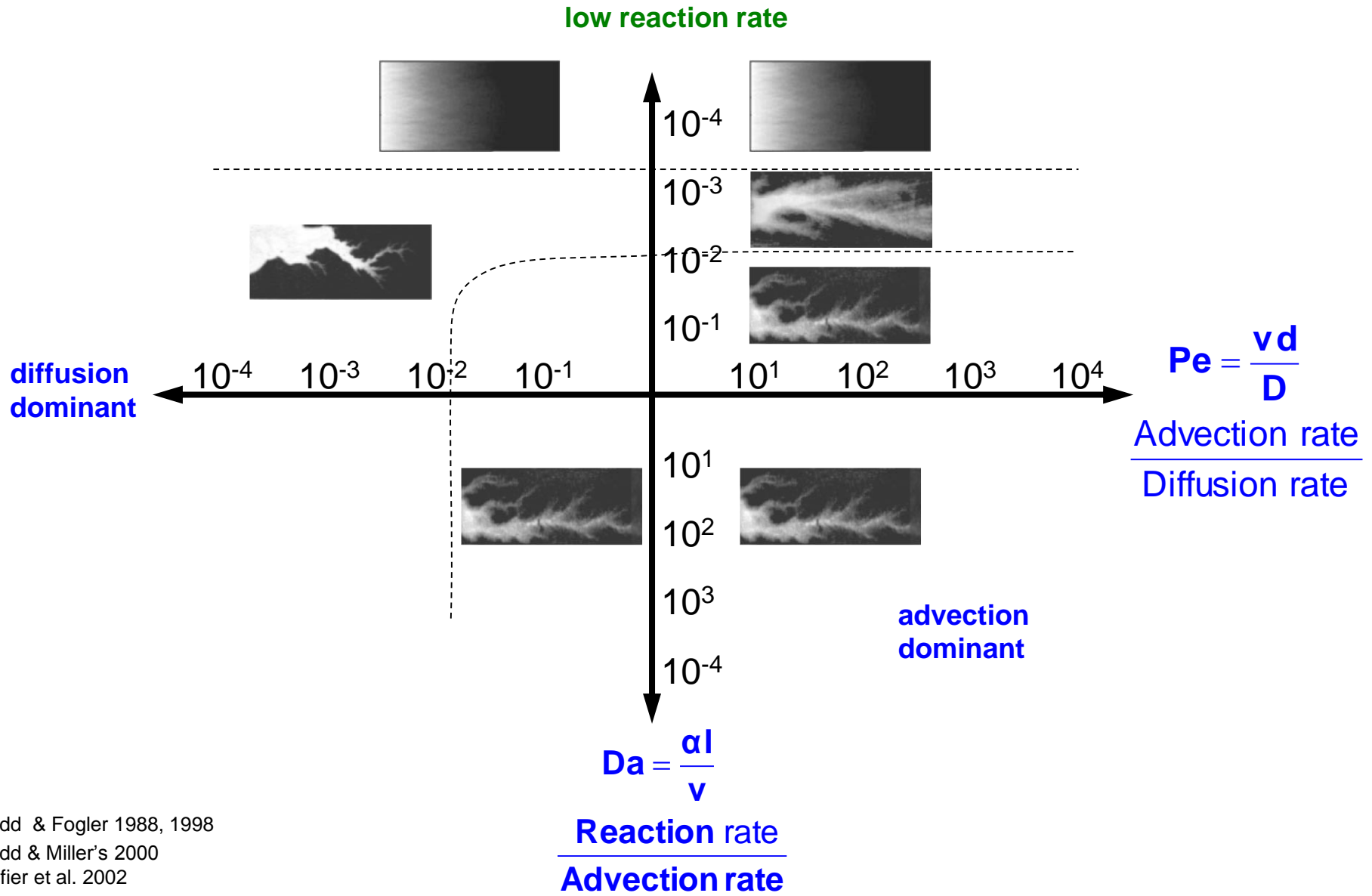
**Permeability**

**Mixed fluids**



National Corvette Museum  
Bowling Green, Kentucky

# Reactive Fluid Transport



Fredd & Fogler 1988, 1998

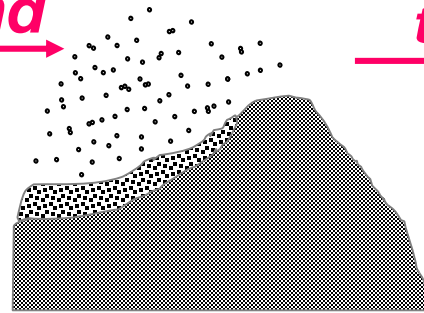
Fredd & Miller's 2000

Golfier et al. 2002

# Volcanic Ash Soils: Formation



*wind* →



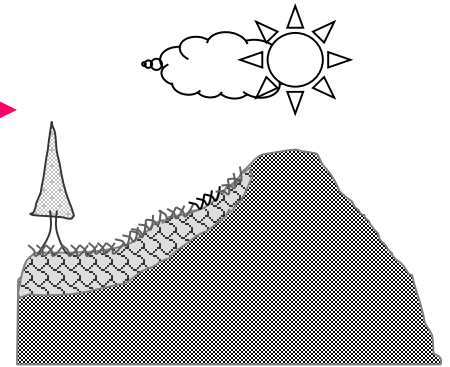
$$e = 0.8-1.5$$

$$S_s \sim 0.1-1 \text{ m}^2/\text{g}$$

volcanic glass

$$k_o = 1 - \sin \phi$$

→ *time*



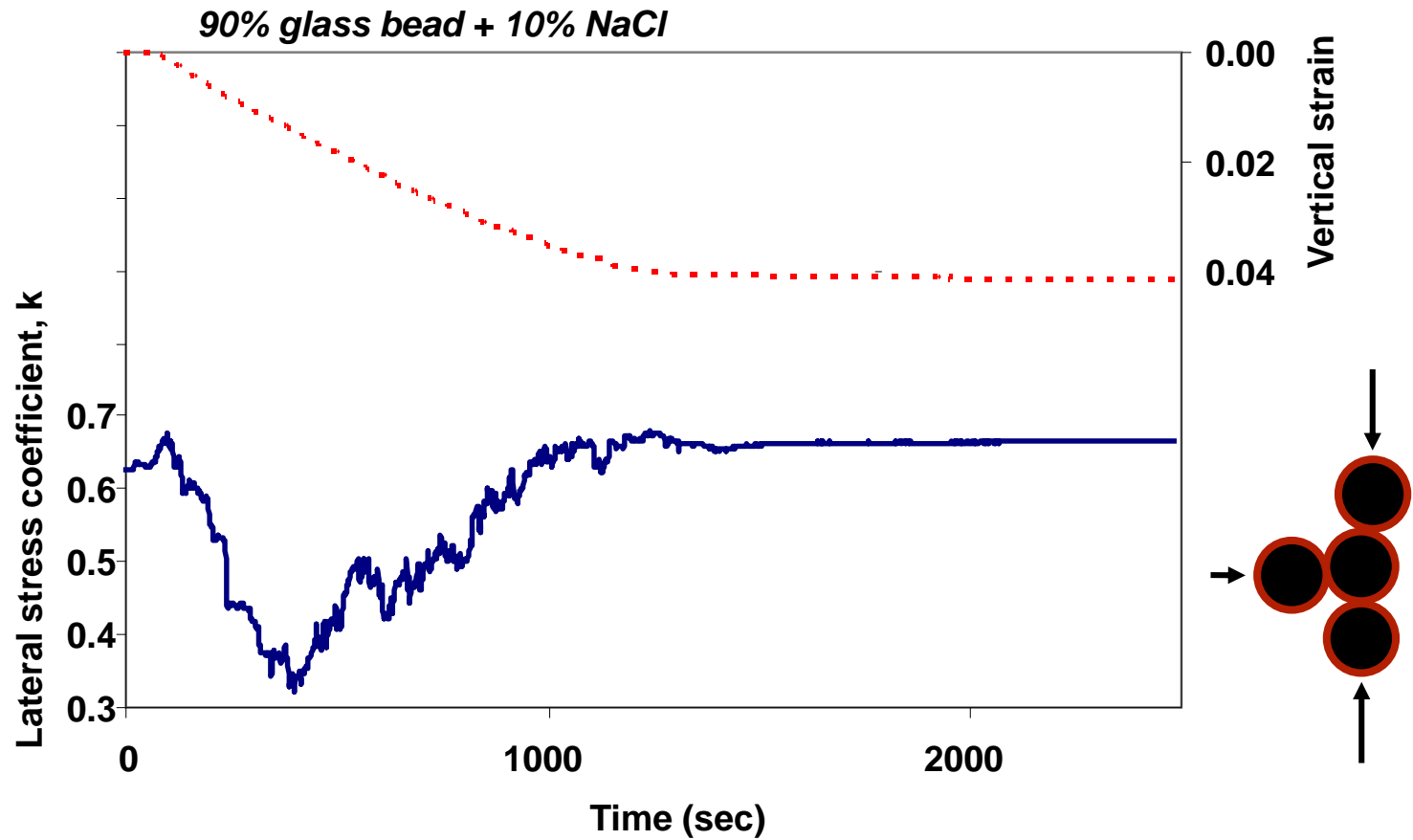
$$e = 2.0-7.0$$

$$S_s = 50\text{-to-}200 \text{ m}^2/\text{g}$$

hallosite  
imogolite  
alophane

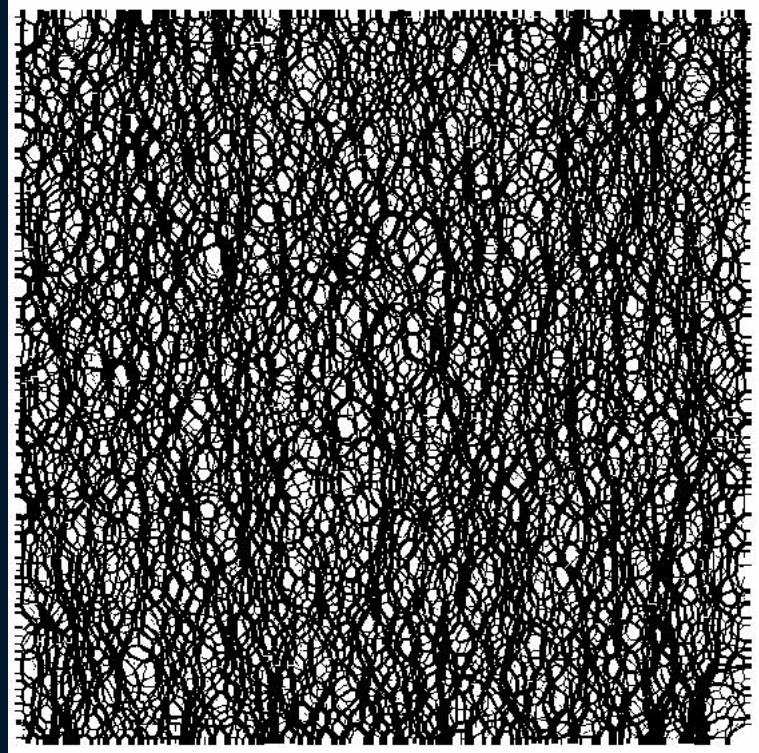
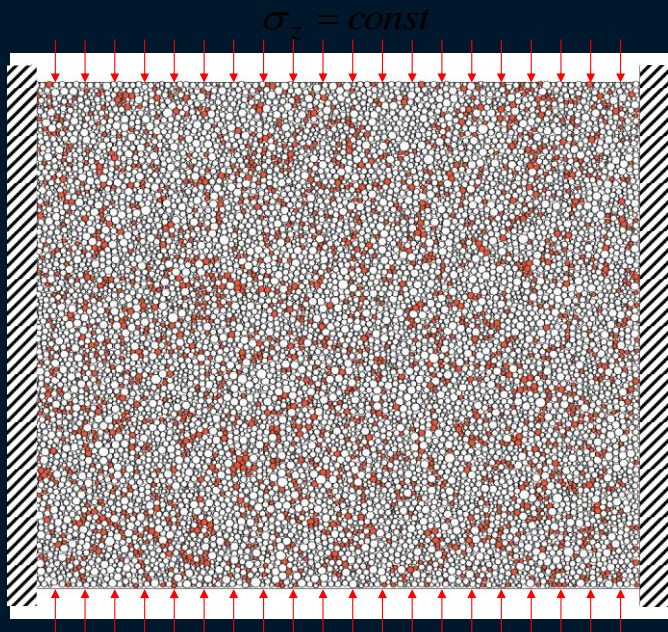
$$k_o = ??$$

# Experimental Results

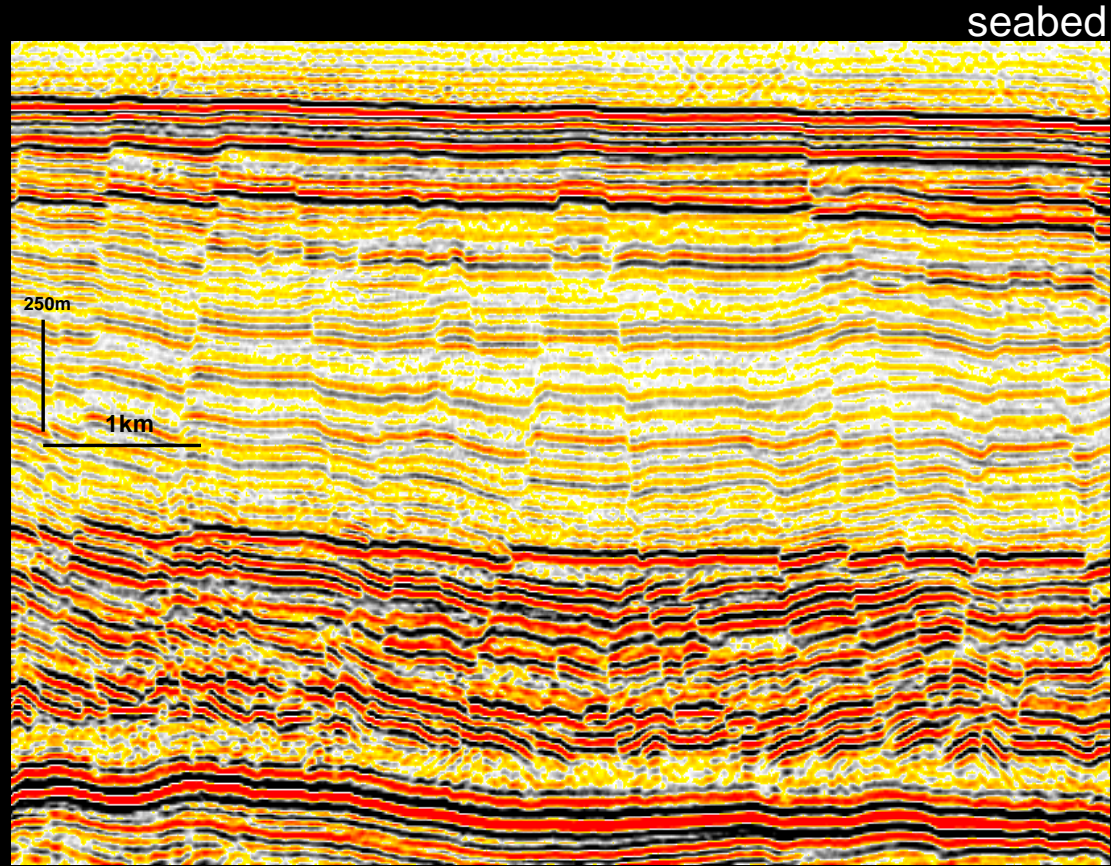




# DEM Simulation $dR/dt=f(N)$

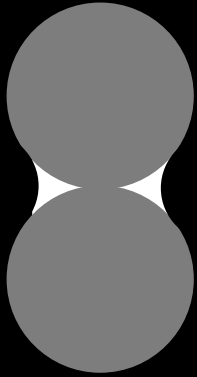


# Natural Sediments

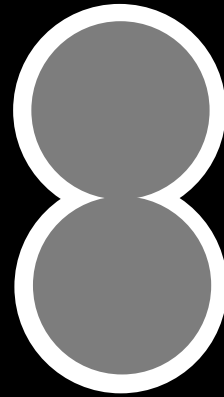


Cartwright (2005)

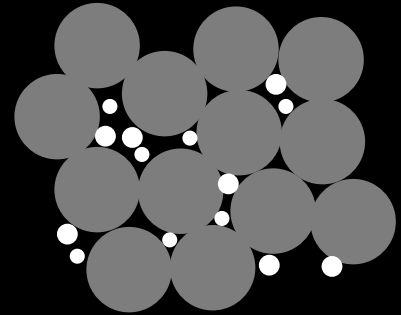
# Precipitation – Pore Habit



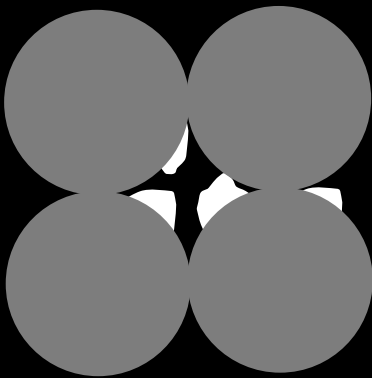
at contact



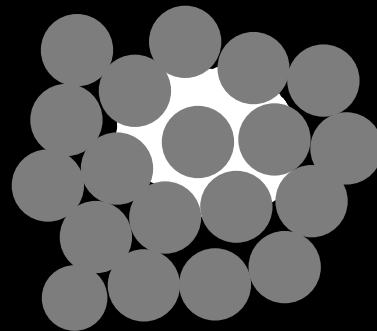
surface coating



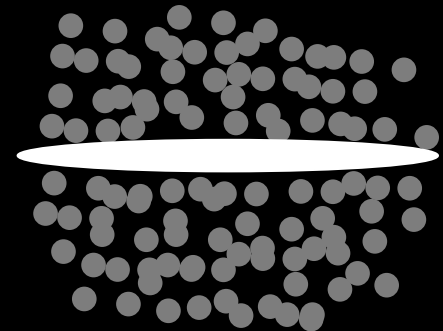
homogeneous nucleation



heterogeneous nucleation

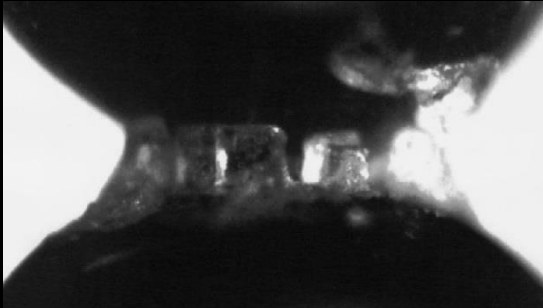


patchy (ripening)

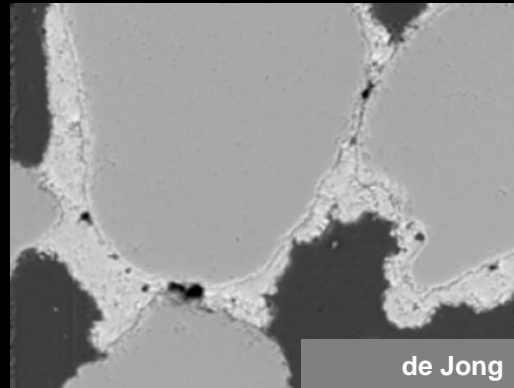


segregated (lenses)

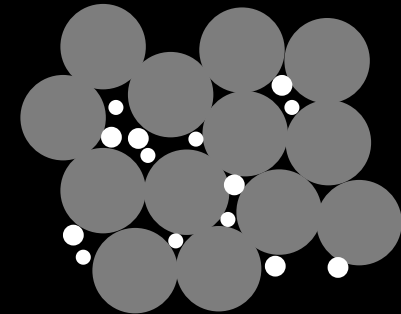
# Precipitation – Pore Habit



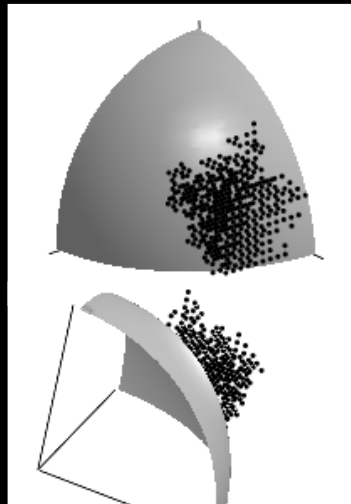
at contact



surface coating



homogeneous nucleation



heterogeneous nucleation



patchy (ripening)



segregated (lenses)

**Formation**

**Size → Forces**

**Shape**

**Soil Classification**

**Diagenesis**

**Shear strength**

**Stiffness**

**Pores**

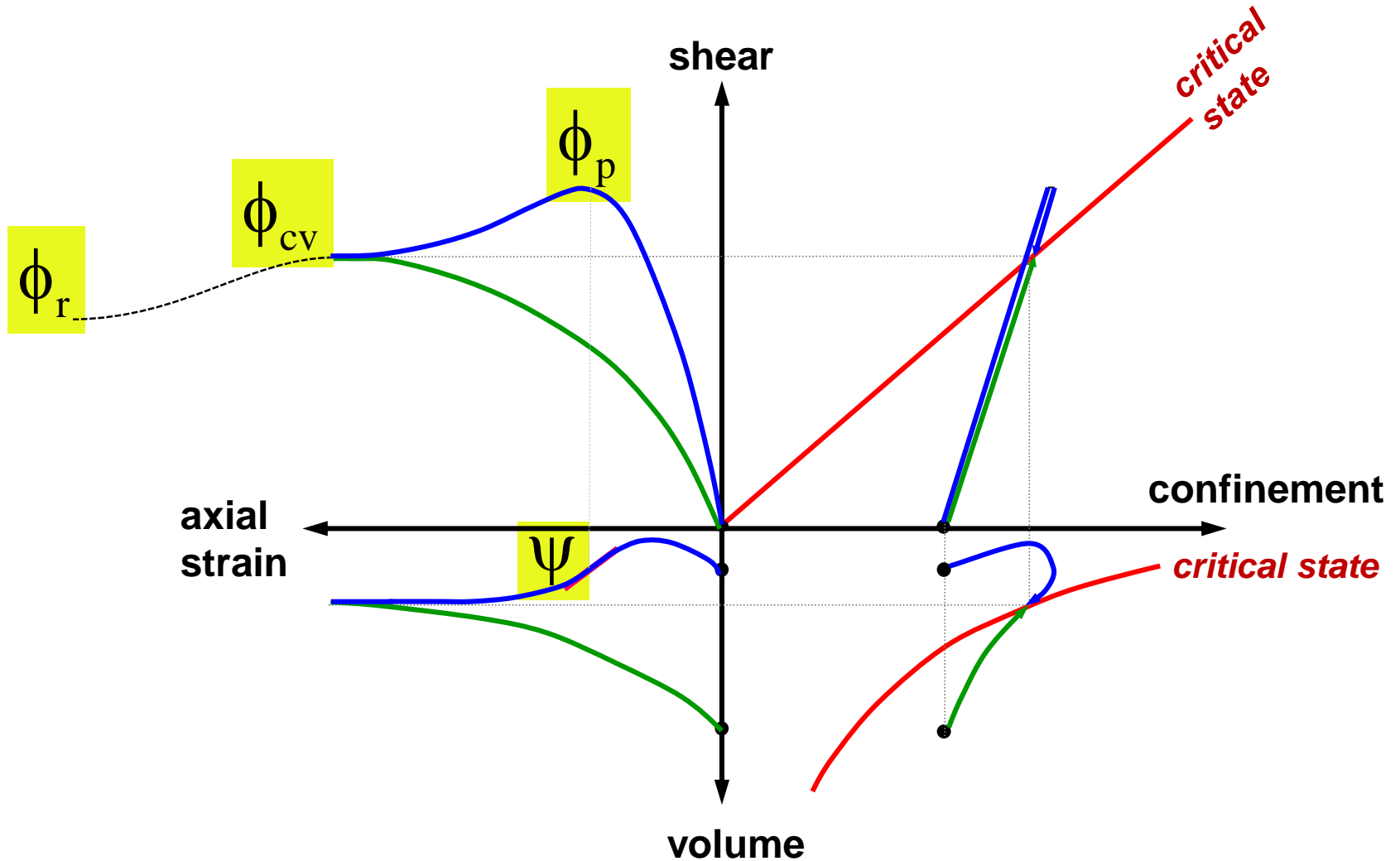
**Permeability**

**Mixed fluids**



coal mine – Australia – [guardian.co.uk](http://guardian.co.uk)

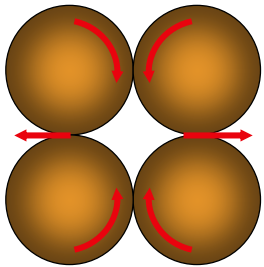
# Sediment Response During Shear



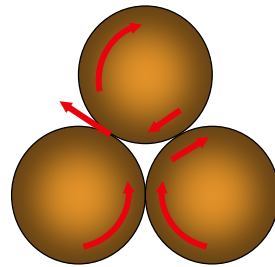
# Constant Volume Shear: “*Critical State*”

Rot. frustration: Coordination  $\downarrow$

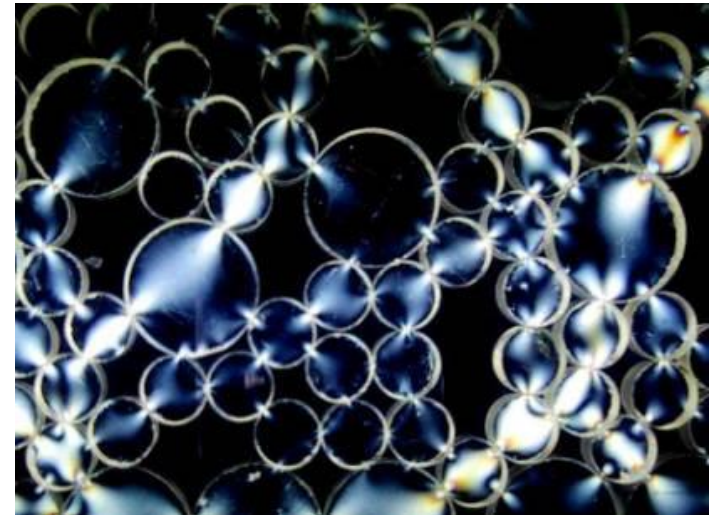
Chain Buckling: Coordination  $\uparrow$



*Free*  
(high  $e$ )

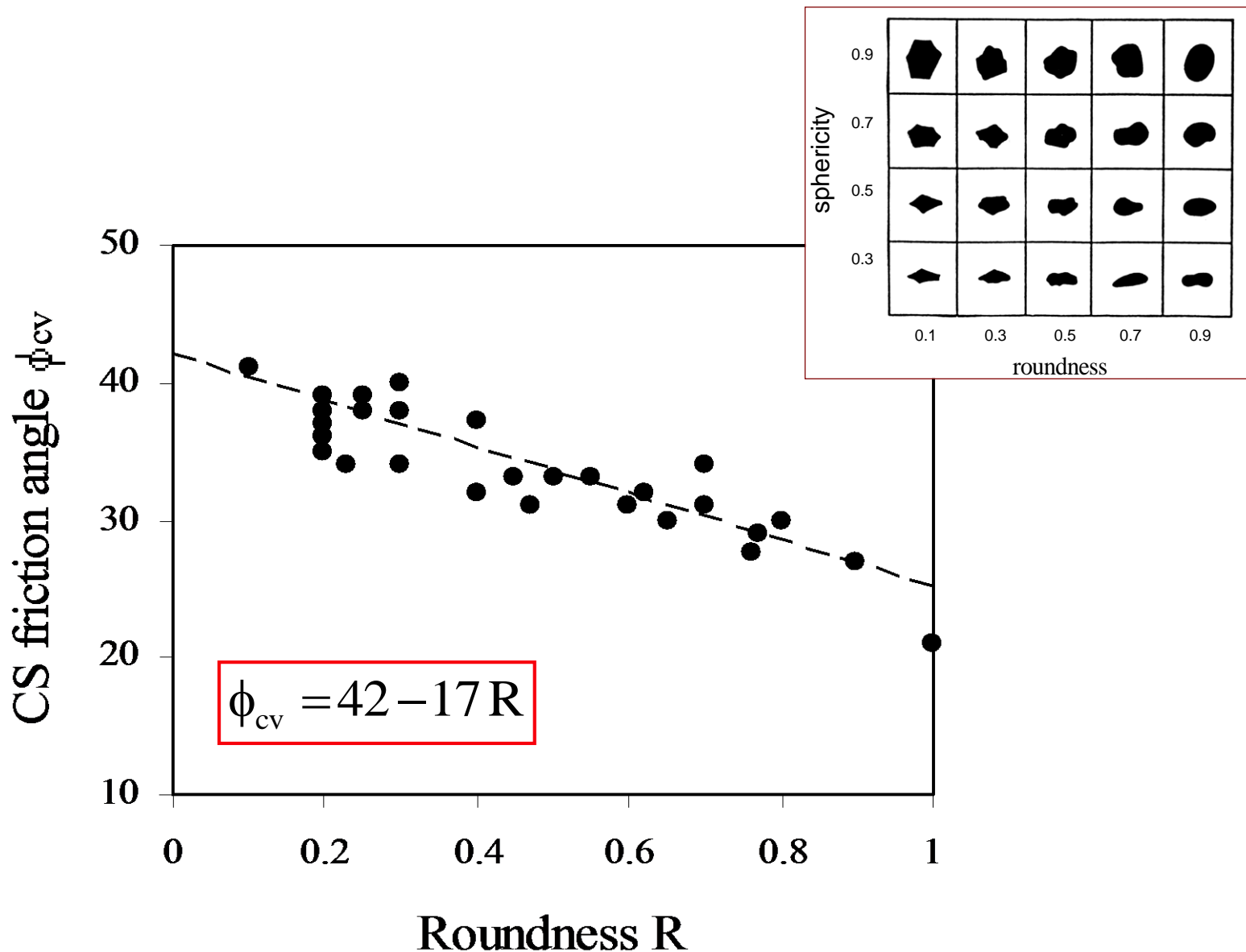


*Frustrated*  
(low  $e$ )



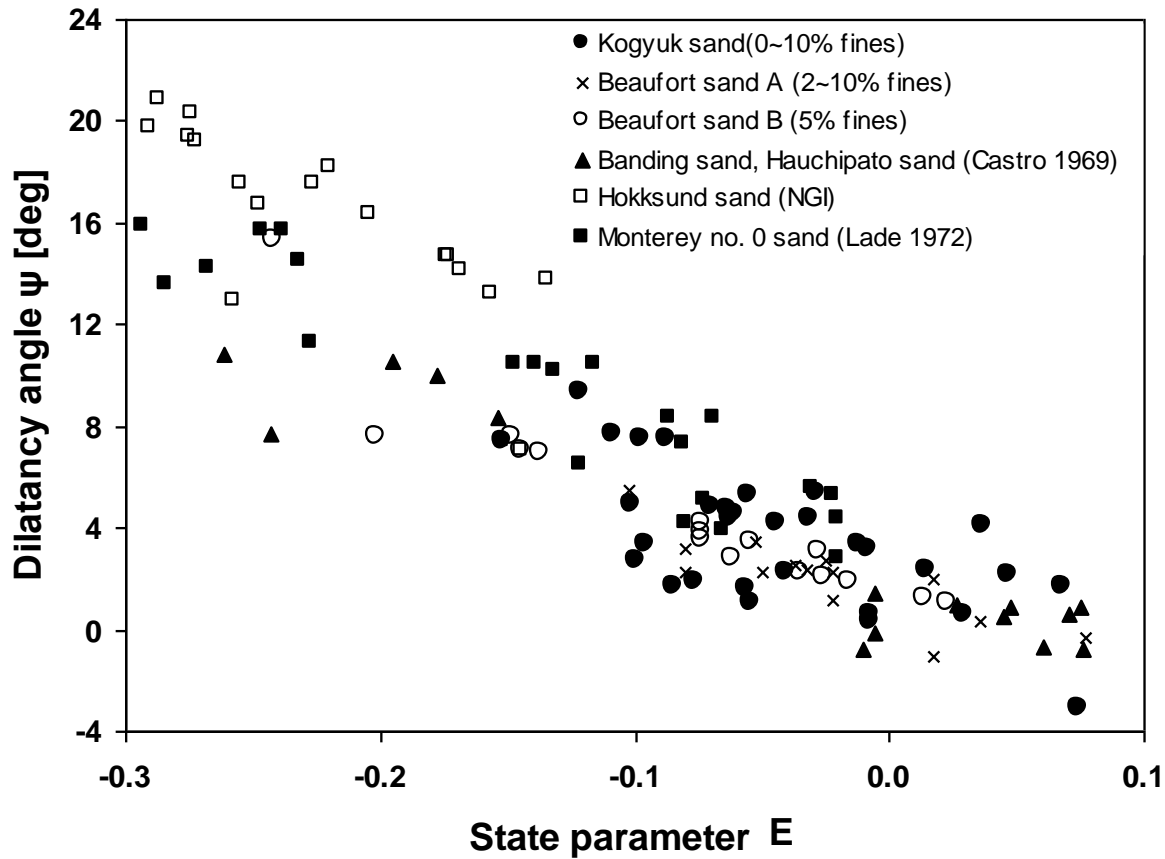
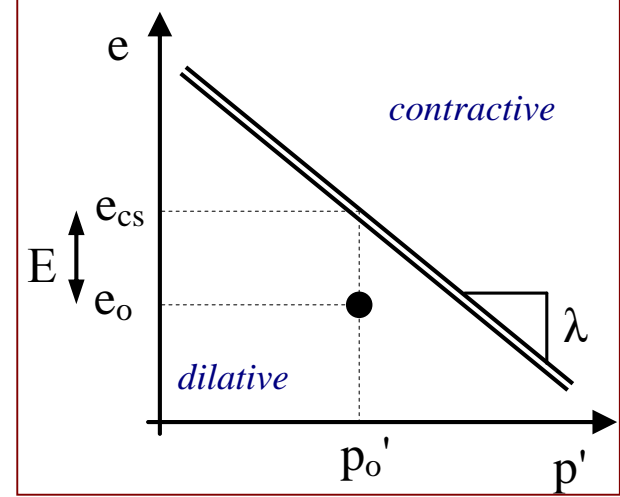
**reduce rotational frustration**  
**avoid contact slip**

# Constant Volume Friction vs. Roundness





# Dilatancy Angle



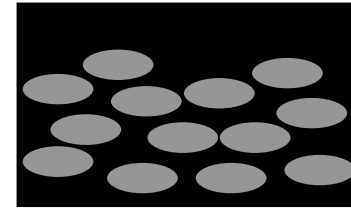
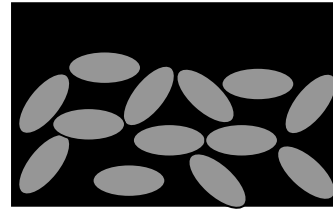
# Peak Friction Angle

**Taylor 1948:**  $\tan \phi_p = \tan \phi_{cv} + \tan \psi$

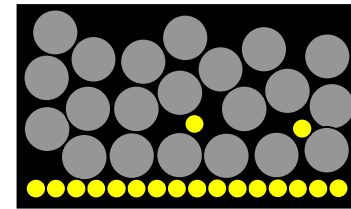
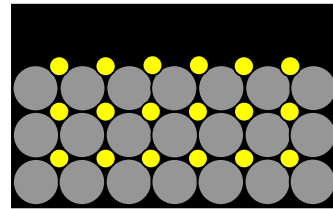
**Bolton 1986:**  $\phi_p = \phi_{cv} + 0.8\psi$

# Residual Friction Angle - very large strains

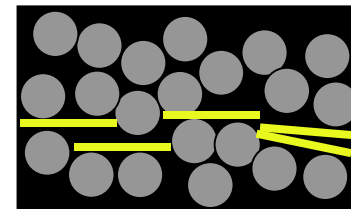
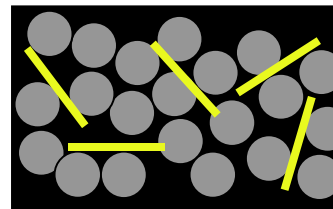
particle alignment



size segregation

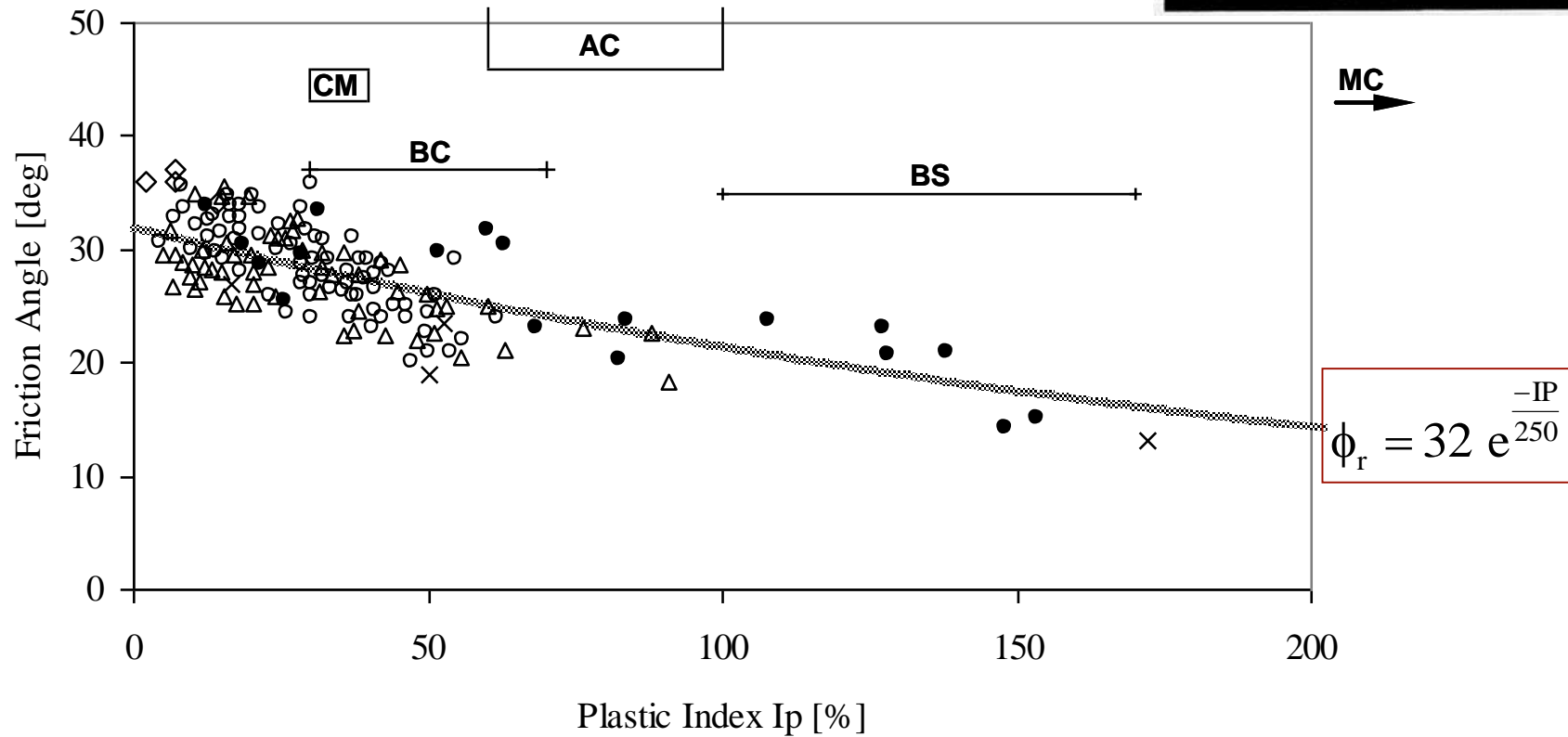
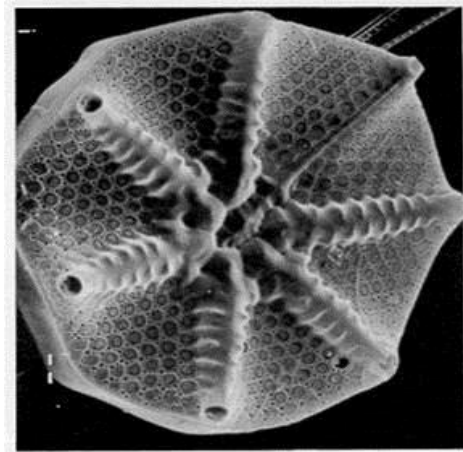


shape segregation



# Residual Friction Angle $\phi_r$

$\phi_r$



Ariake Clay "AC"

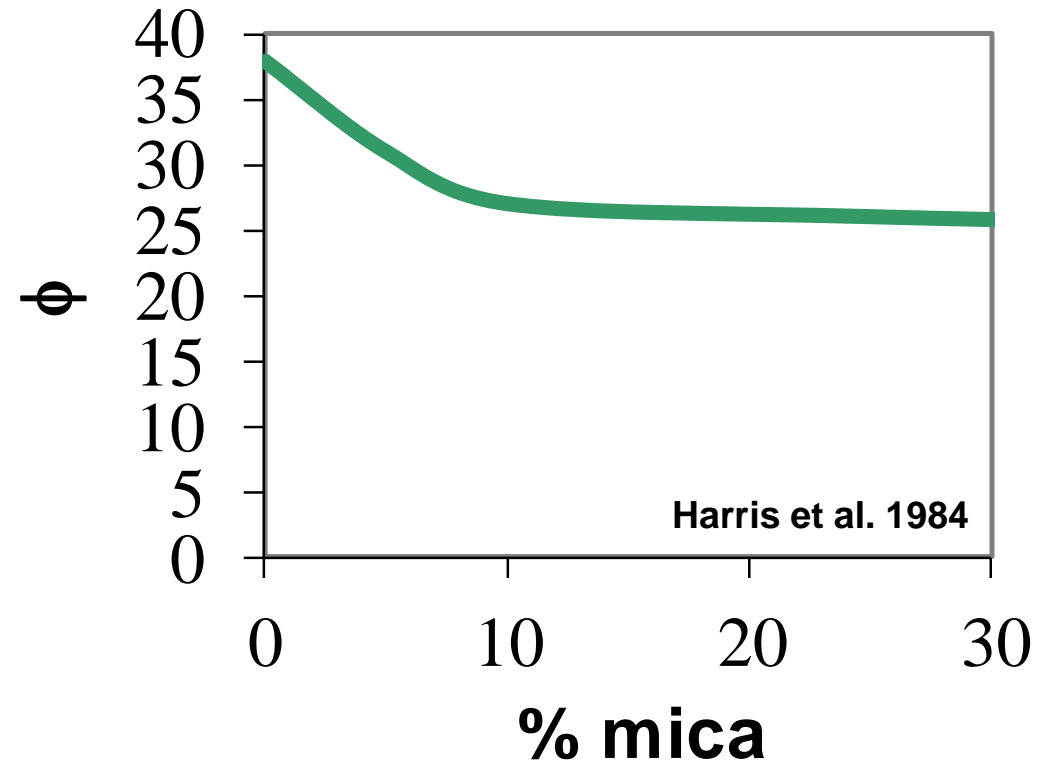
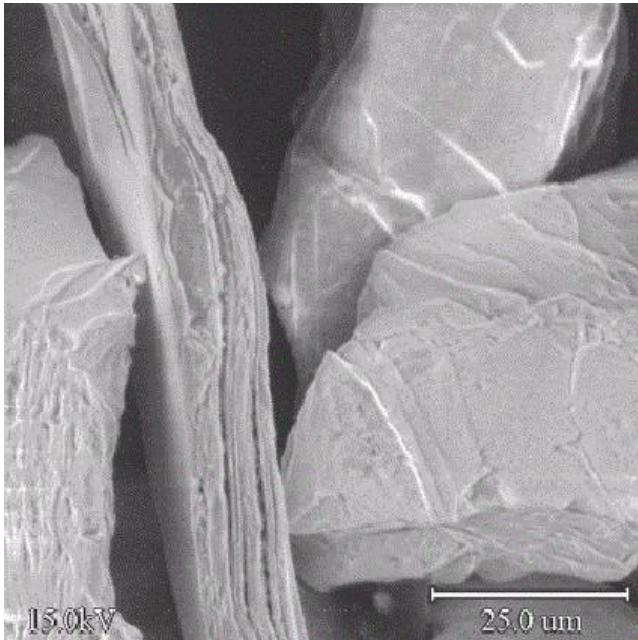
Bangkok Clay "BC"

Bogota Soil "BS"

Cooper Marl "CM"

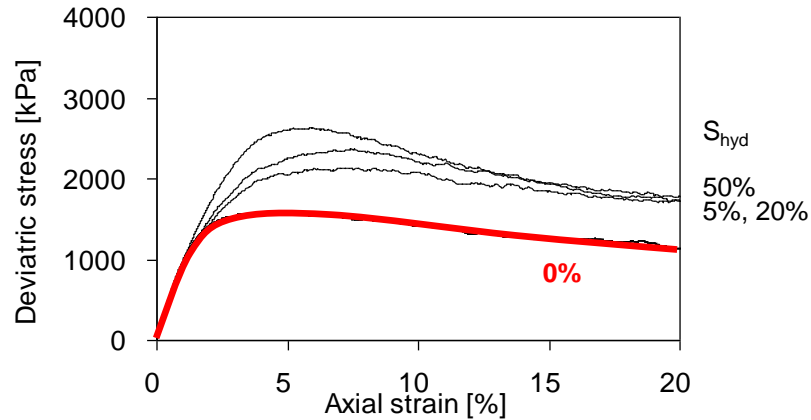
Mexico City "MC"

# Platy particles

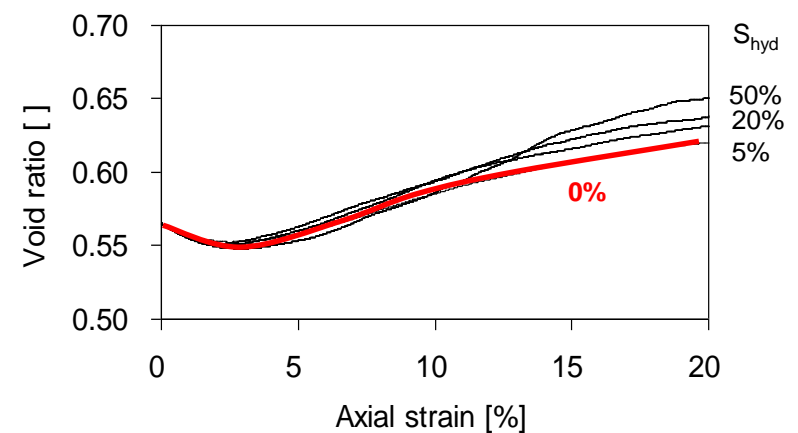
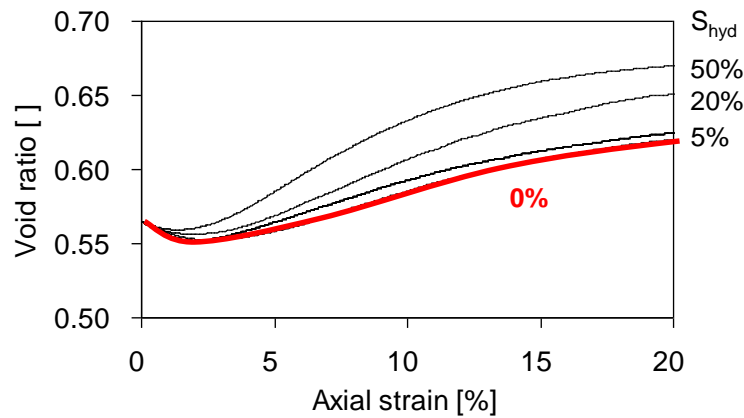
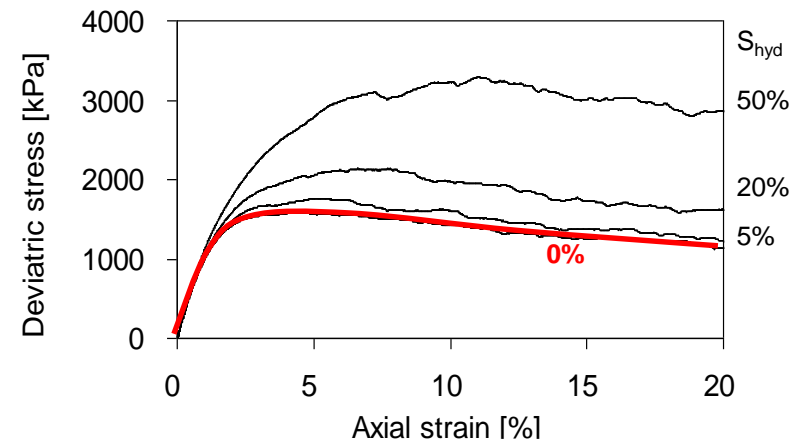


# Precipitation → ?

*distributed cementation*



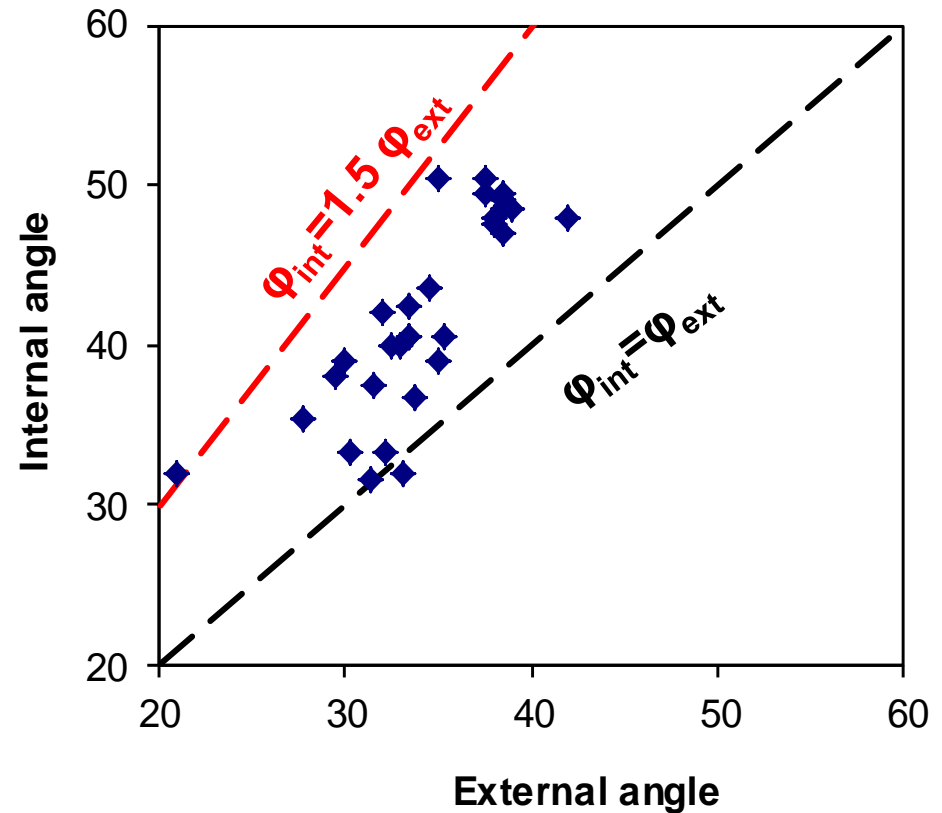
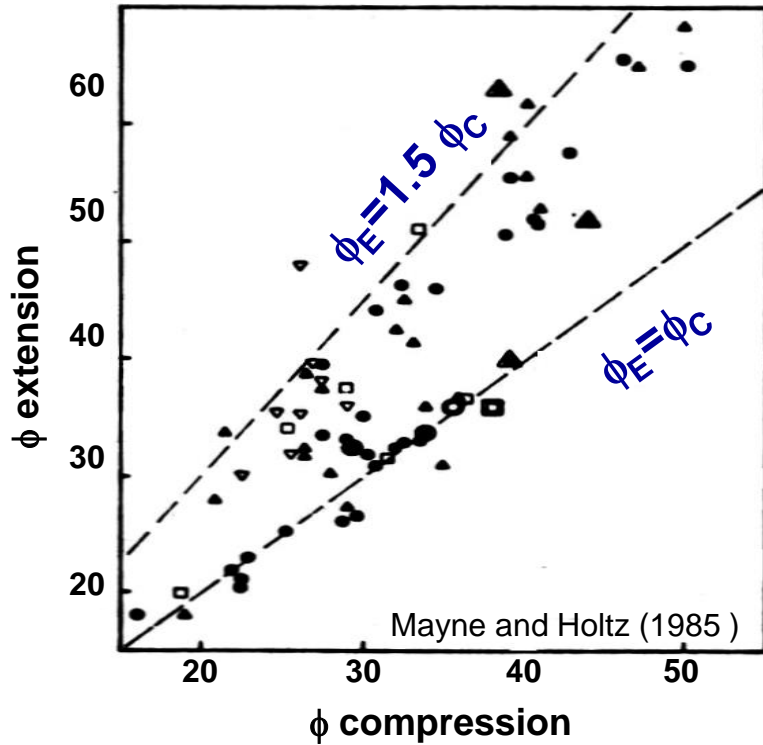
*patchy cementation*



**Note: increase in stiffness , strength, dilation with  $S_{hyd}$  - pore habit affect dilation**

# Frictional strength anisotropy

$$\phi_E = 1.0 \text{ to } 1.5 \phi_C$$



**Formation**

**Size → Forces**

**Shape**

**Soil Classification**

**Diagenesis**

**Shear strength**

**Stiffness**

**Pores**

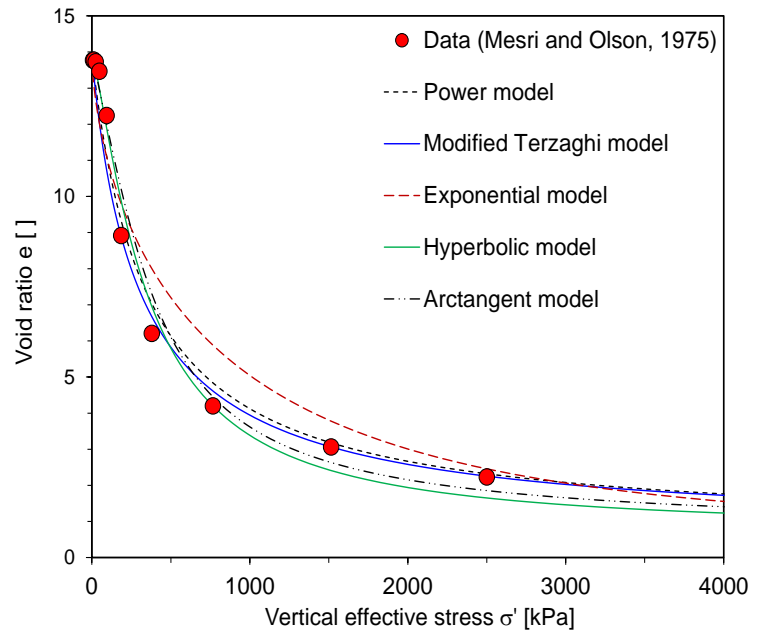
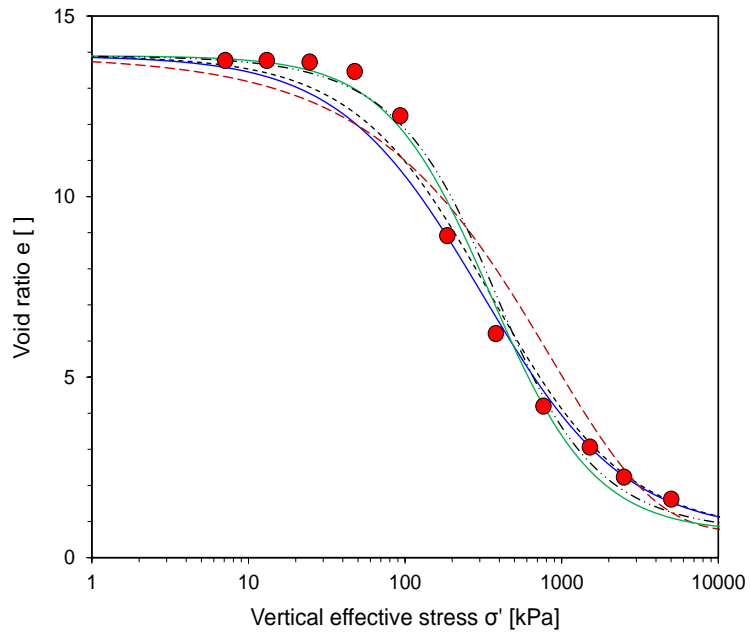
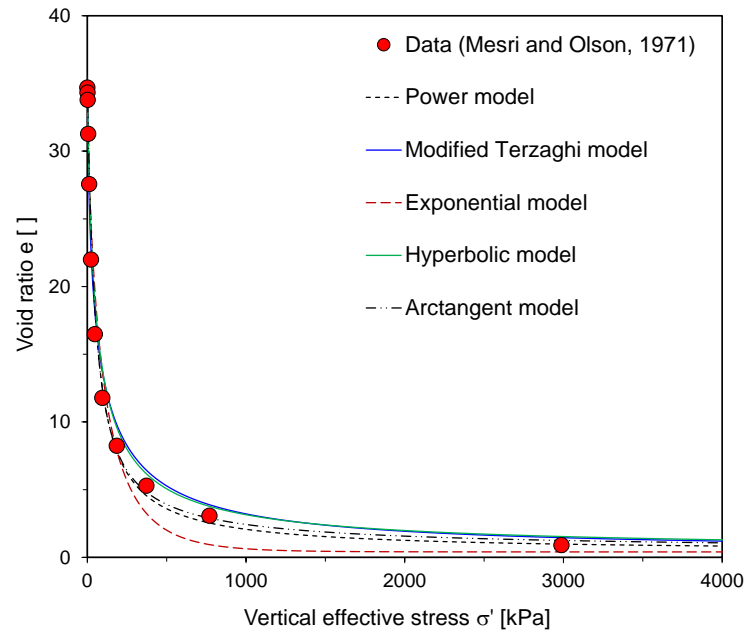
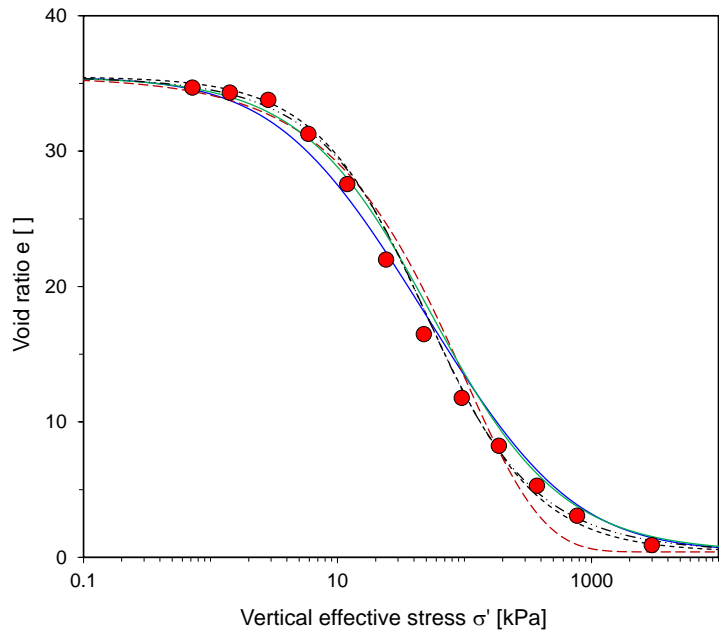
**Permeability**

**Mixed fluids**

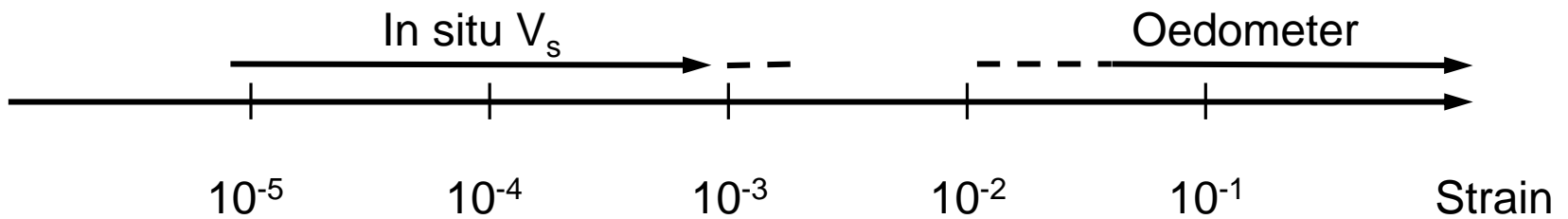
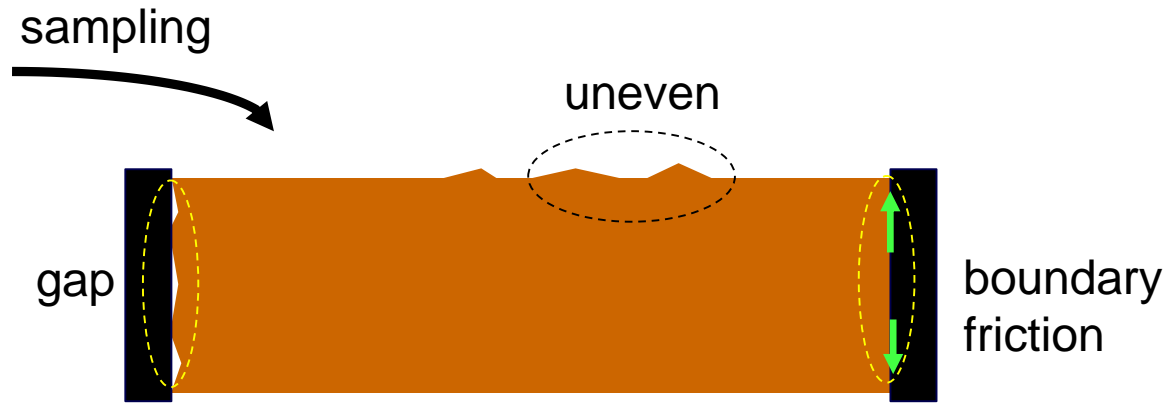




<b>Semi-log</b>	<p><i>Classical (1<sup>st</sup> order)</i></p> <p>Terzaghi &amp; Peck (1948) Schofield &amp; Wroth (1968)</p>	$e = e_{1\text{kPa}} - C_c \log\left(\frac{\sigma'}{1\text{kPa}}\right)$
	<p><i>Cubic (3<sup>rd</sup> order)</i> <i>(high-stress correction)</i></p> <p>Burland (1990)</p>	$e = e_{\text{ref}} - \alpha \cdot \log\left(\frac{\sigma'}{\sigma'_{\text{ref}}}\right) + \beta \cdot \left[\log\left(\frac{\sigma'}{\sigma'_{\text{ref}}}\right)\right]^3$
	<p><i>Modified</i> <i>(low and high stress asymptotes)</i></p>	$e = e_c - C_c \log\left(\frac{1\text{kPa}}{\sigma' + \sigma'_L} + \frac{1\text{kPa}}{\sigma'_H}\right)^{-1}$
<b>Power</b>	<p><i>From gas to soil</i></p> <p>Hansen (1969); Butterfield (1979); Juárez-Badillo (1981); Houlsby &amp; Wroth, (1991); Pestana &amp; Whittle (1995)</p>	$e = e_H + (e_L - e_H) \left(\frac{\sigma' + \sigma'_c}{\sigma'_c}\right)^{-\beta}$
<b>Exponential</b>	<p><i>Gompertz function</i> <i>(classical exp: <math>\beta=1</math>)</i></p> <p>Gregory et al. (2006) Cargill (1984 – <math>\beta=1</math>)</p>	$e = e_H + (e_L - e_H) \cdot \exp\left[-\left(\frac{\sigma'}{\sigma'_c}\right)^\beta\right]$
<b>Hyperbolic</b>	<p><i>Ramberg-Osgood</i> <i>(classical hyperbolic: <math>\beta=1</math>)</i></p>	$e = e_L - (e_L - e_H) \frac{1}{1 + \left(\frac{\sigma'}{\sigma'_c}\right)^\beta}$
<b>Arctangent</b>	<p><i>S-shaped function</i></p> <p>G. Goldsztein</p>	$e = e_L + \frac{2}{\pi} (e_L - e_H) \arctan\left[-\left(\frac{\sigma'}{\sigma'_c}\right)^\beta\right]$



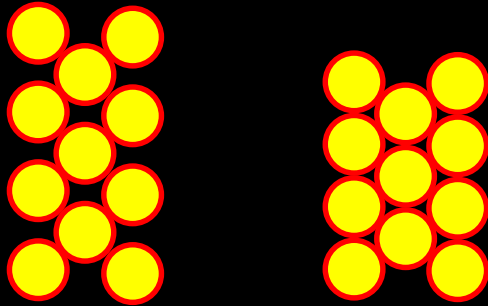
# Oedometer?



$$\frac{\Delta e}{1+e_o} = C_c \log \left( \frac{\sigma'_o + \Delta \sigma'}{\sigma'_o} \right)$$

# Small Strain Stiffness

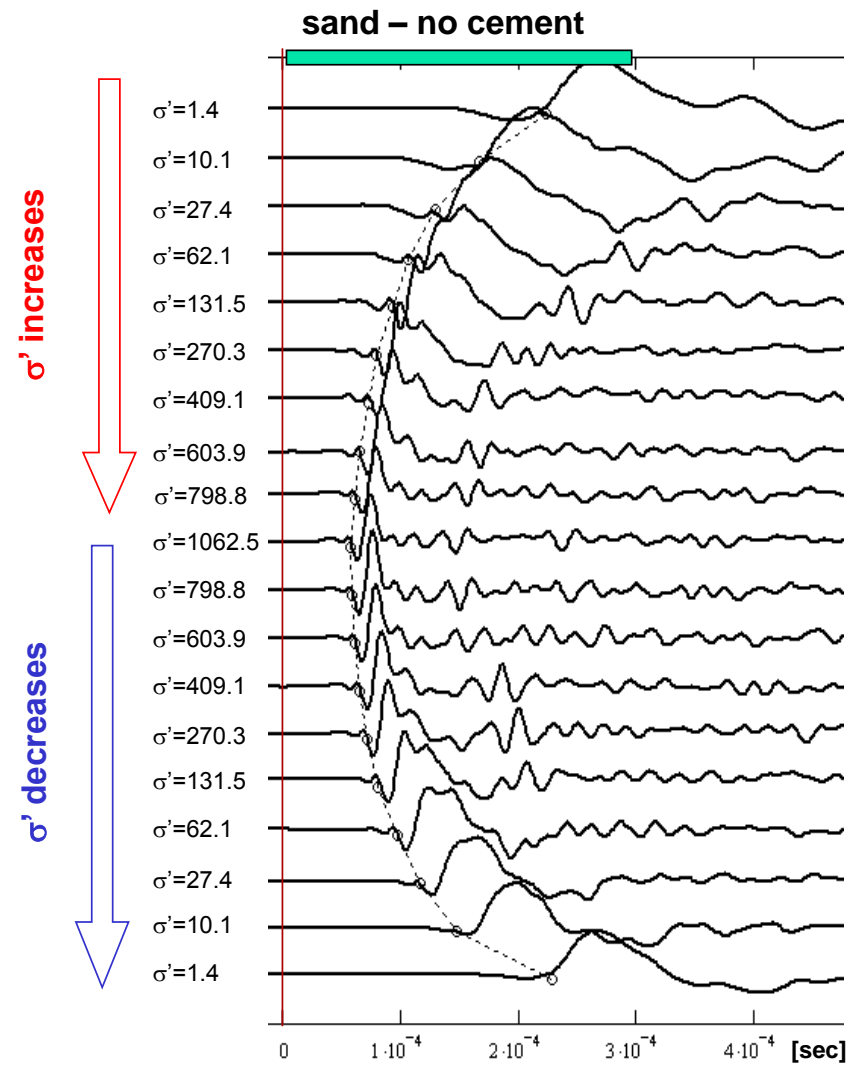
## 1: Fabric change



## 2: Contact deformation

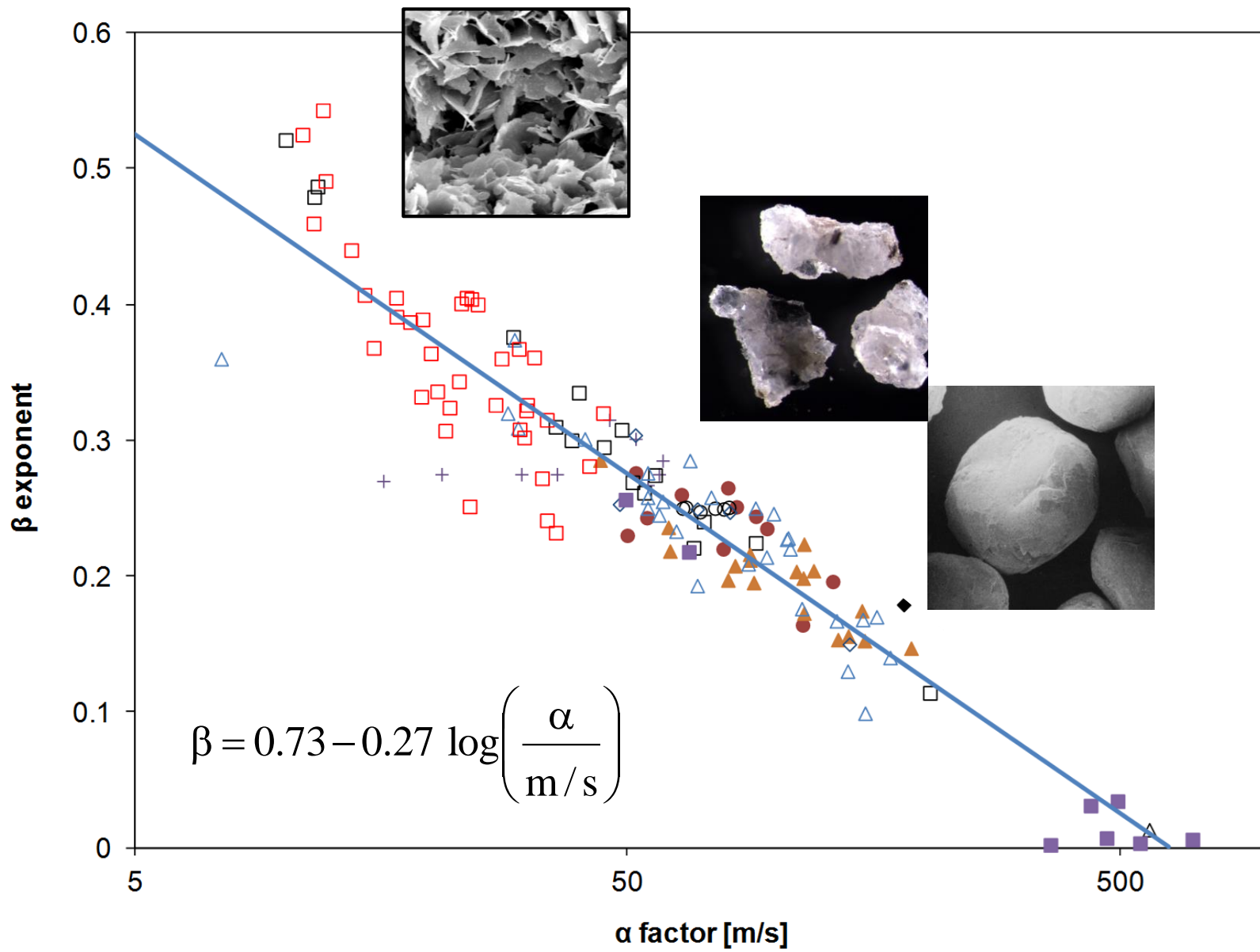


Frocht

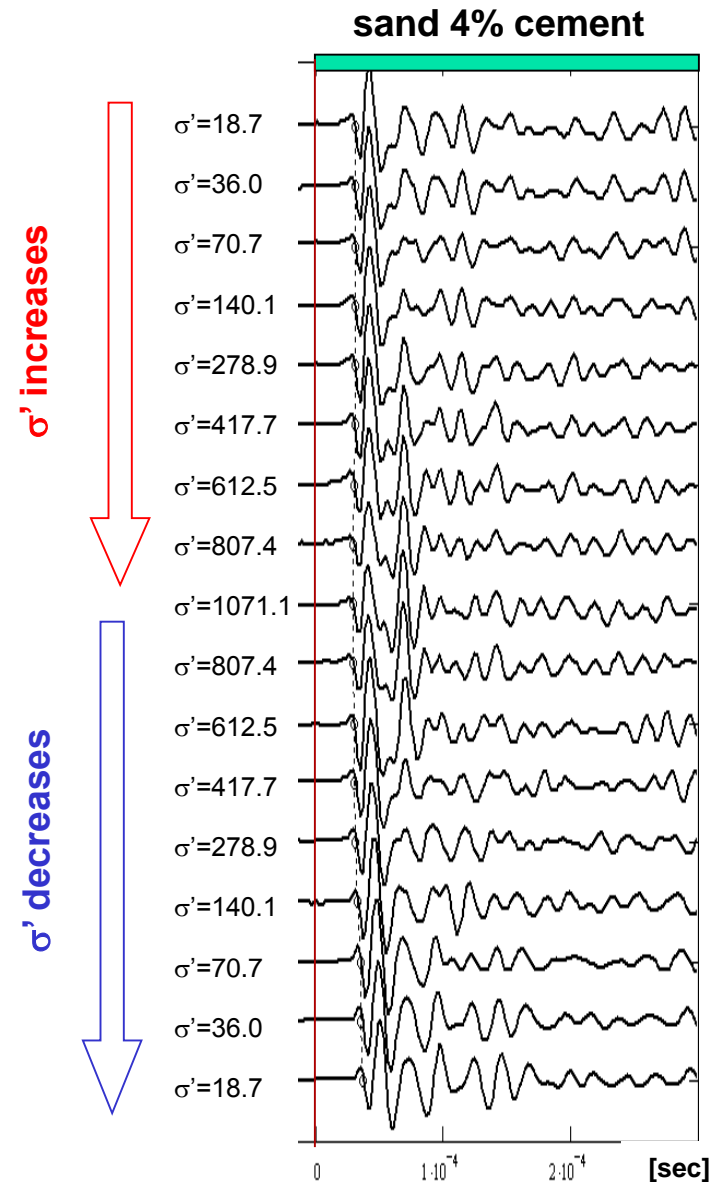
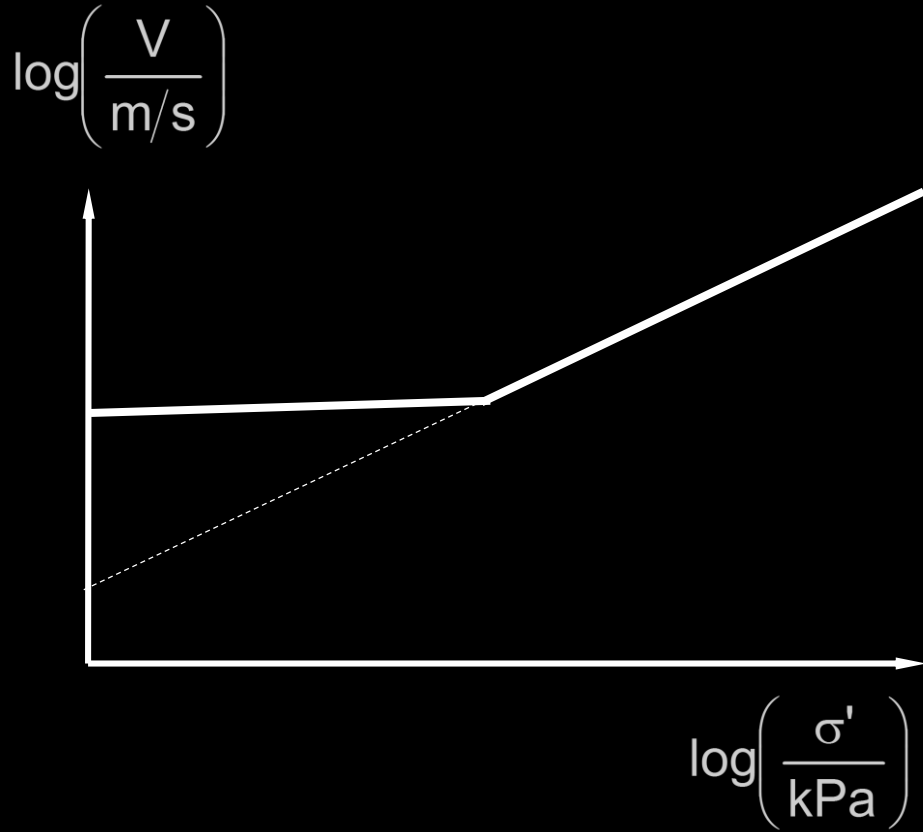


# Velocity-Stress: **Contact + Fabric**

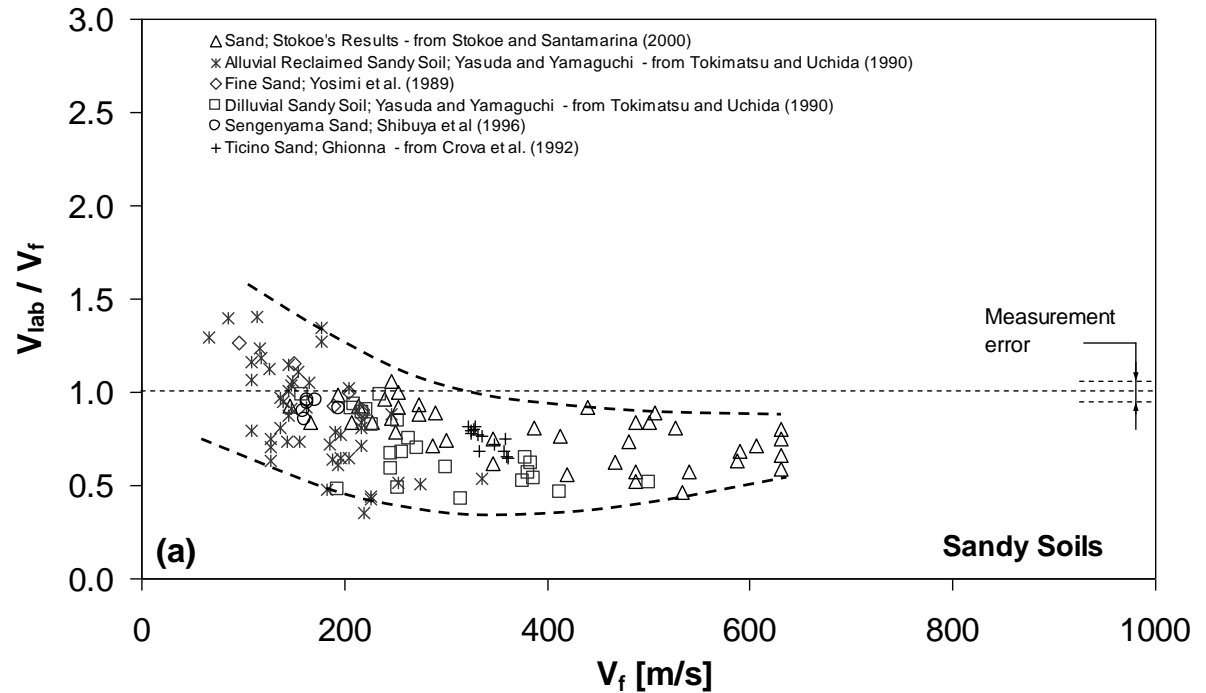
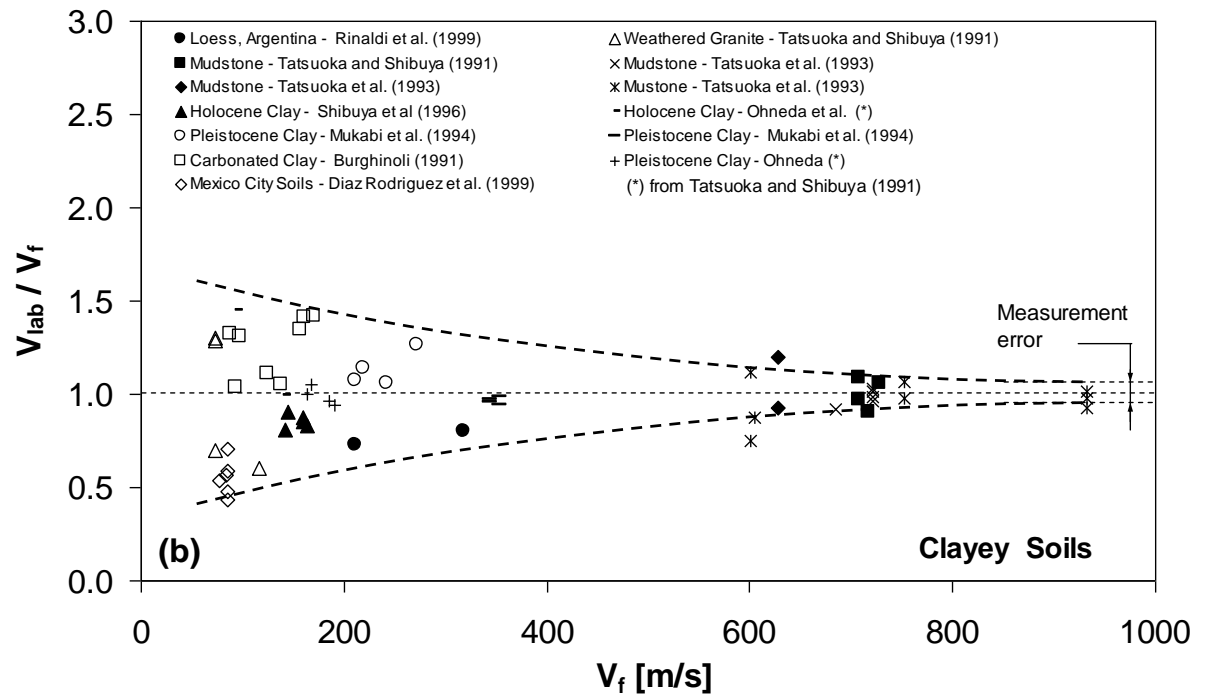
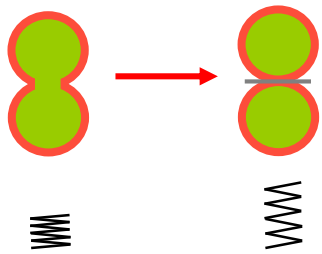
$$V_s = \alpha \left( \frac{\sigma'_x + \sigma'_y}{2P_a} \right)^\beta$$



# Cementation Controlled Stiffness



# Sampling effect



**Formation**

**Size → Forces**

**Shape**

**Soil Classification**

**Diagenesis**

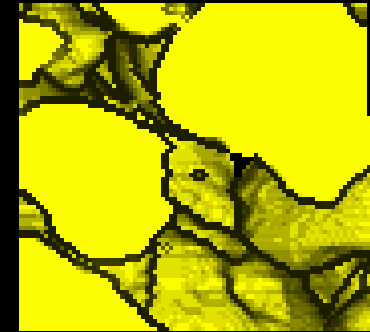
**Shear strength**

**Stiffness**

**Pores**

**Permeability**

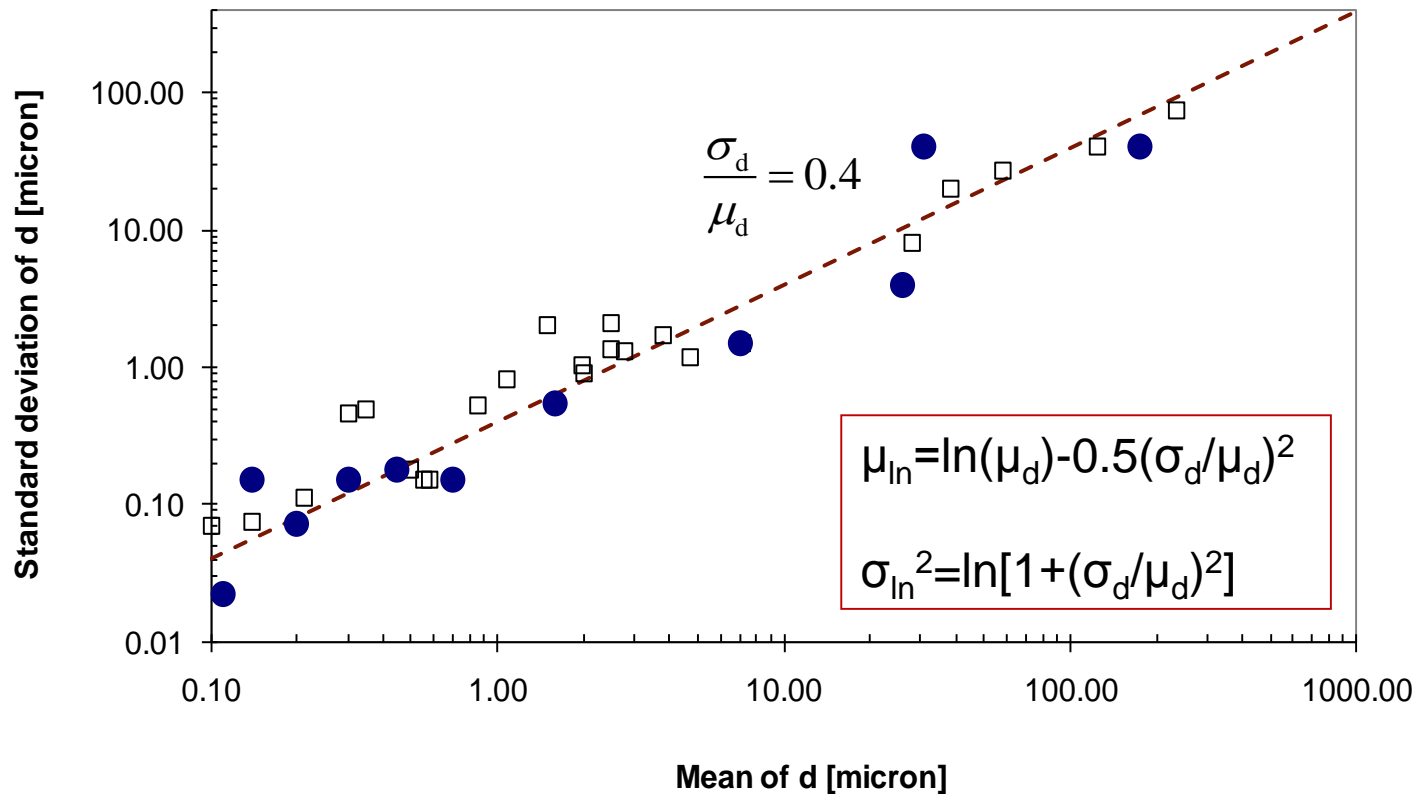
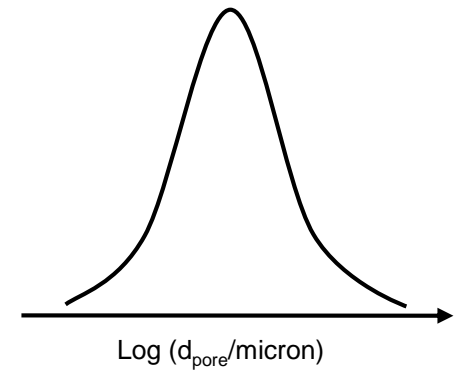
**Mixed fluids**



D. Frost

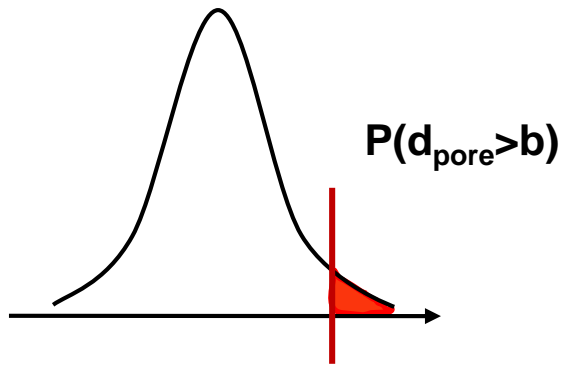
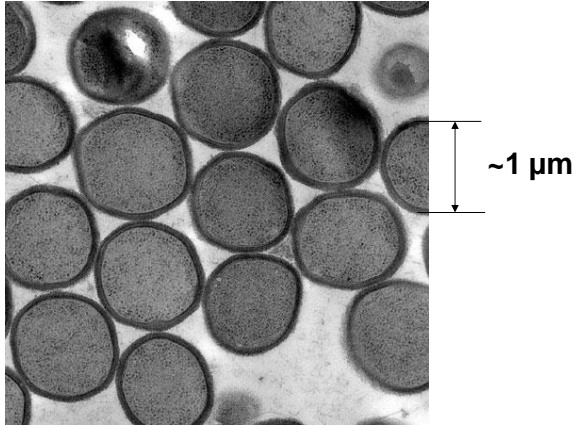


# Pore Size Distribution

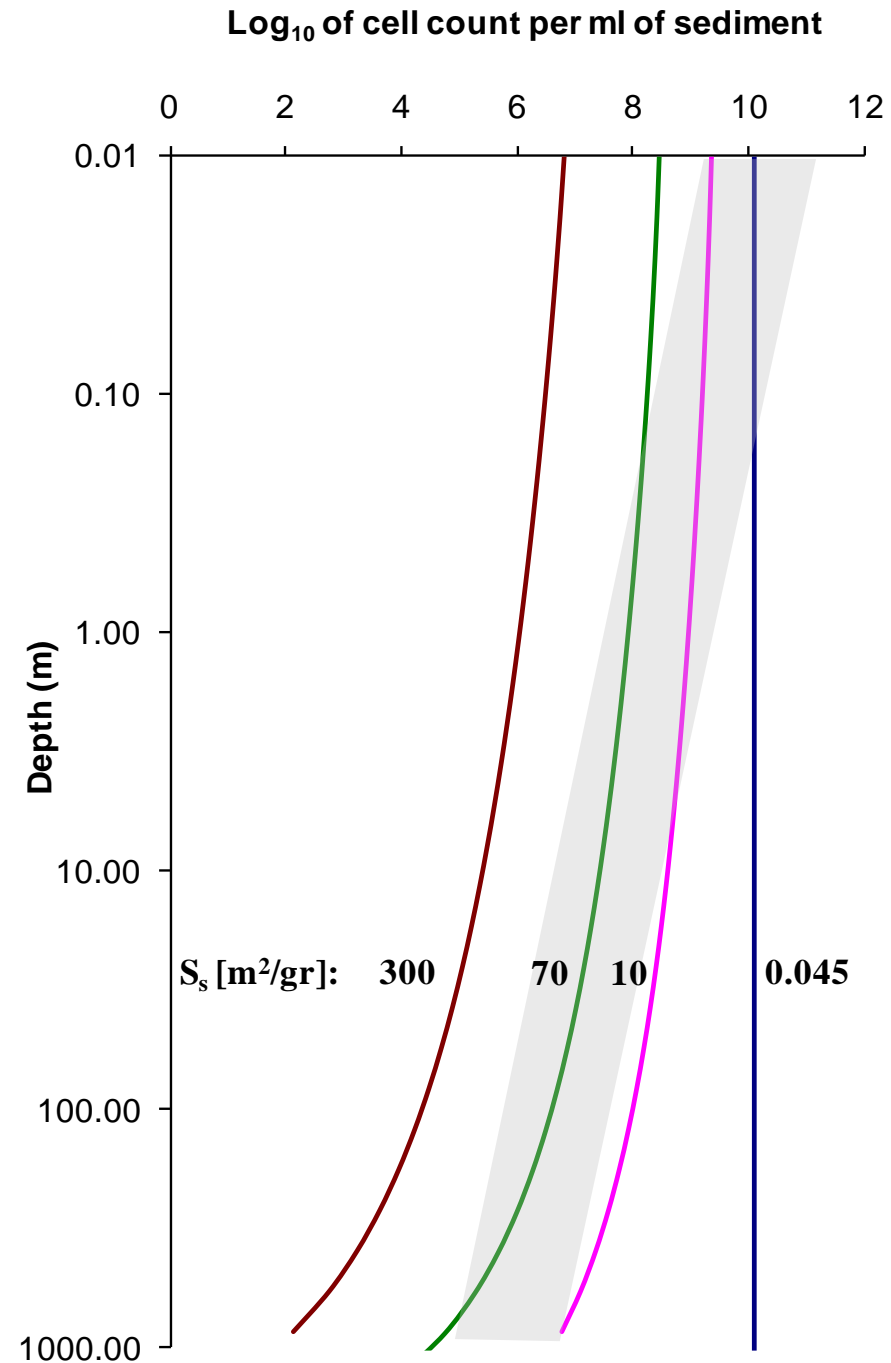


# Bioactivity?

*Bacillus subtilis*



$$c_{\text{sed}} = c_{\text{fl}} \cdot n \cdot P(d \geq b)$$



**Formation**

**Size → Forces**

**Shape**

**Soil Classification**

**Diagenesis**

**Shear strength**

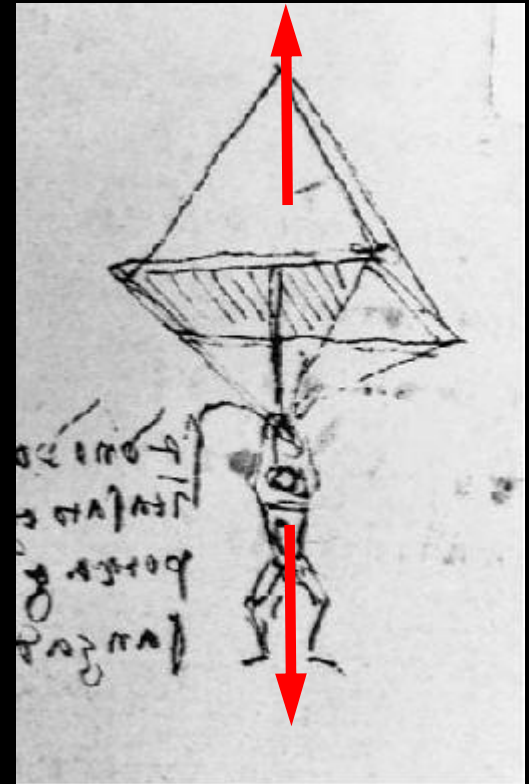
**Stiffness**

**Pores**

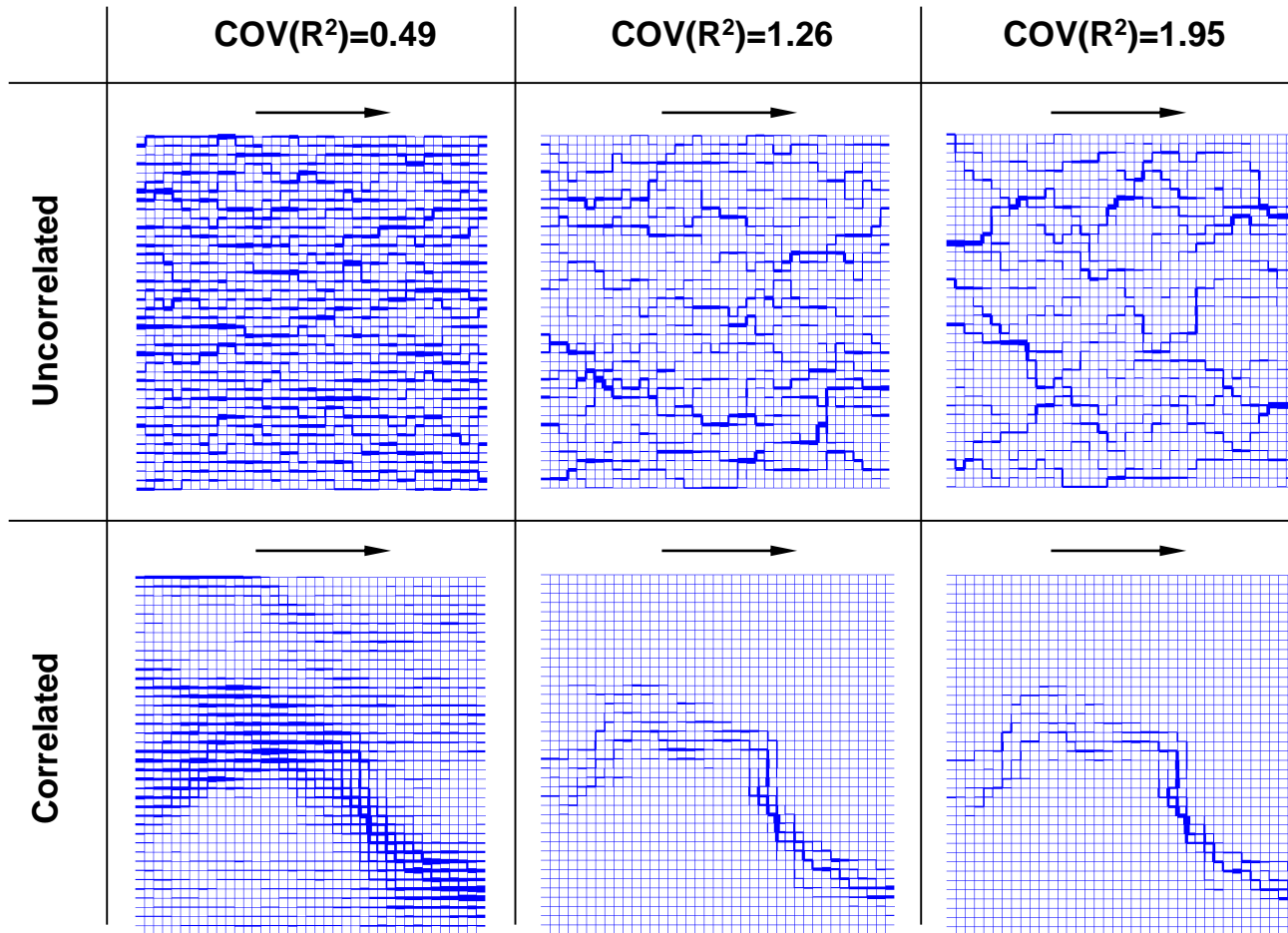
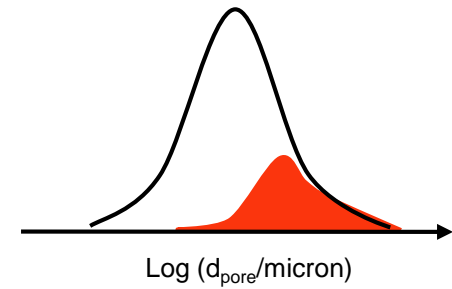
**Permeability**

**Mixed fluids**

Leonardo's Parachute

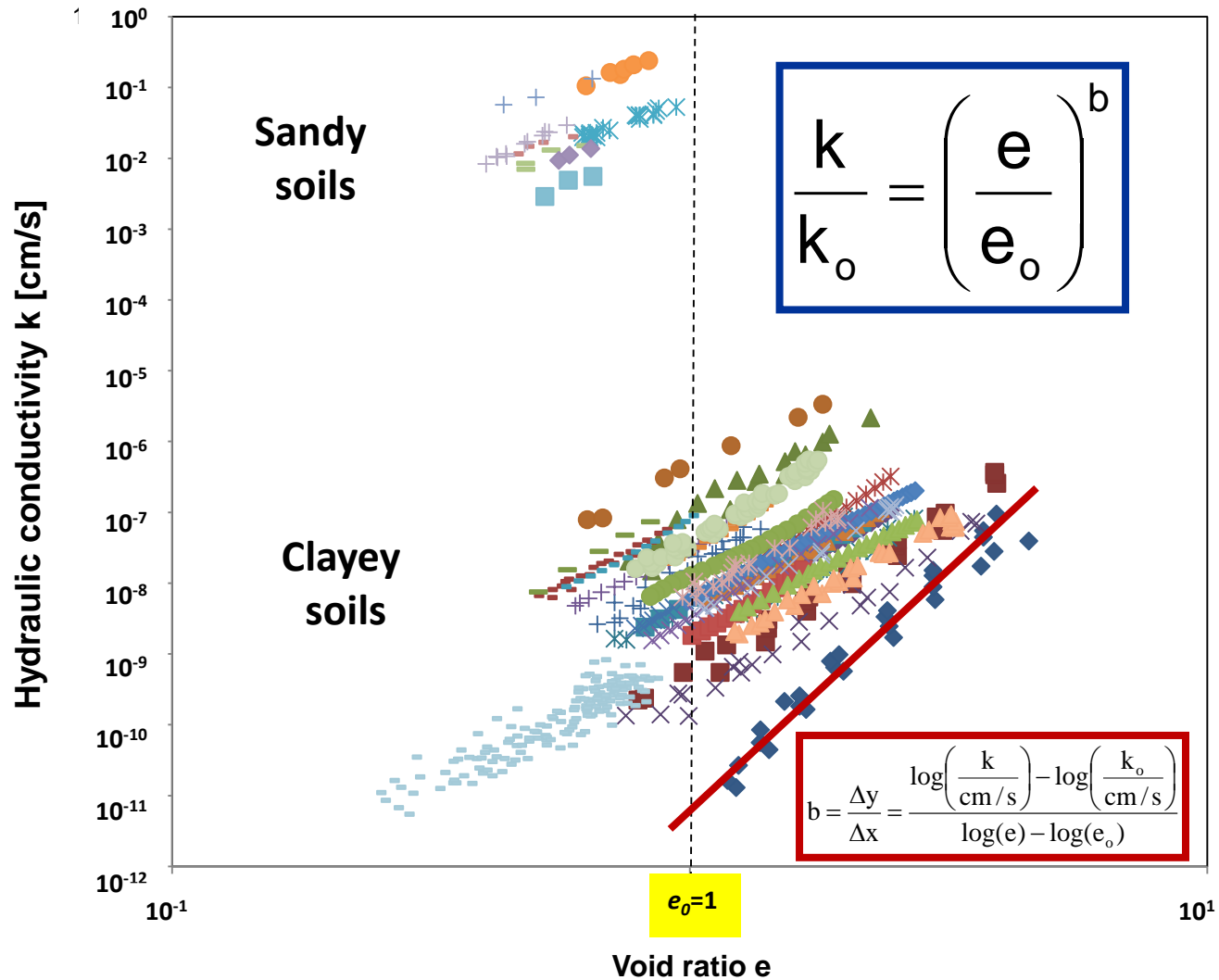


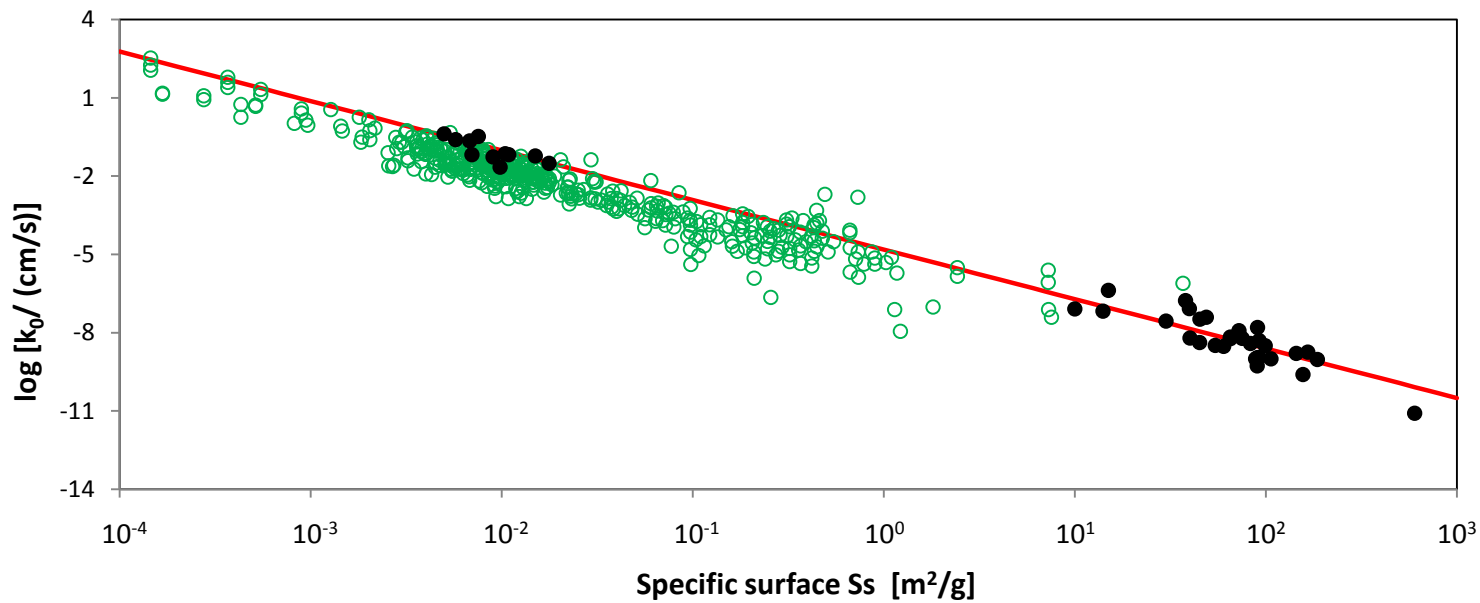
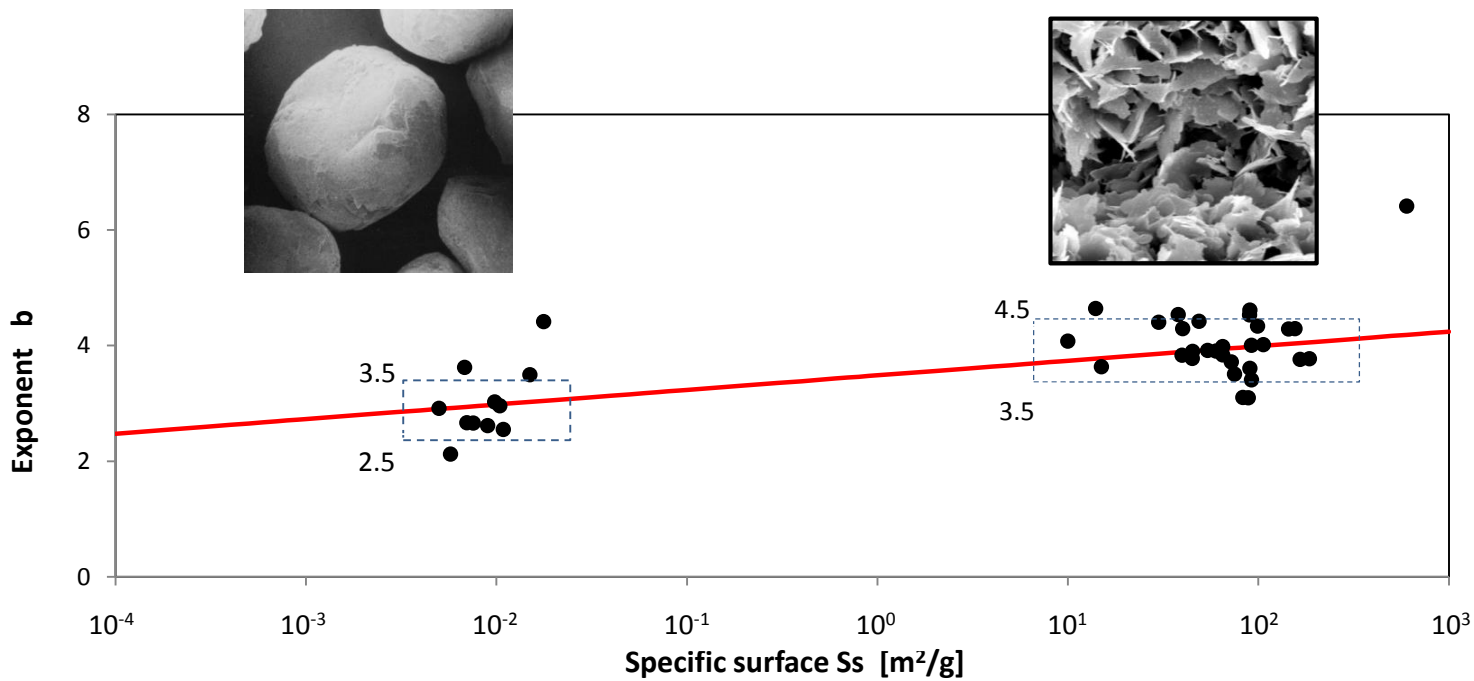
# Pore Network (Poiseuille flow)



*preferential flow along interconnected large pores*

# Permeability during compression





**Formation**

**Size → Forces**

**Shape**

**Soil Classification**

**Diagenesis**

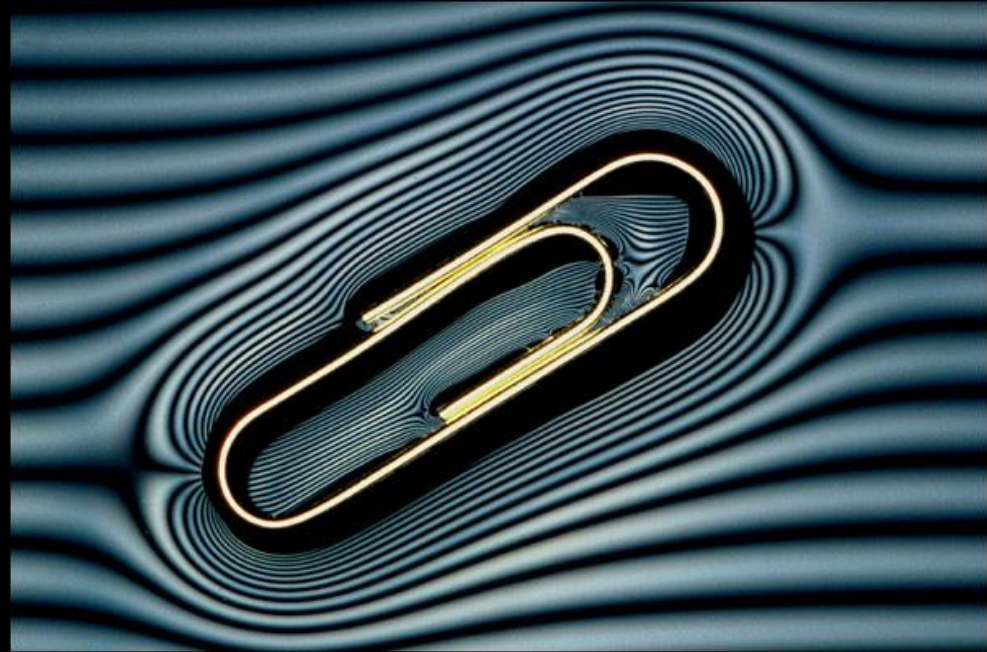
**Shear strength**

**Stiffness**

**Pores**

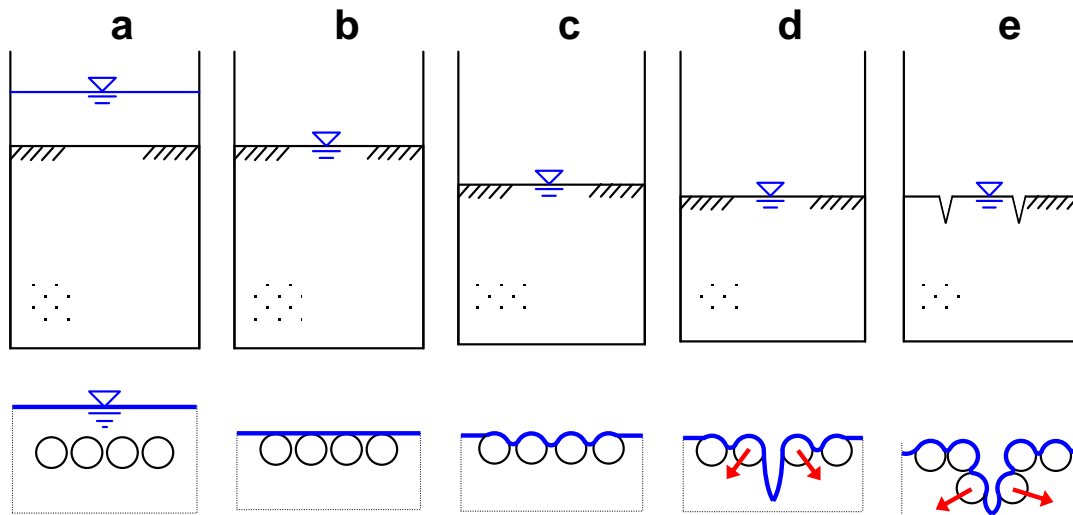
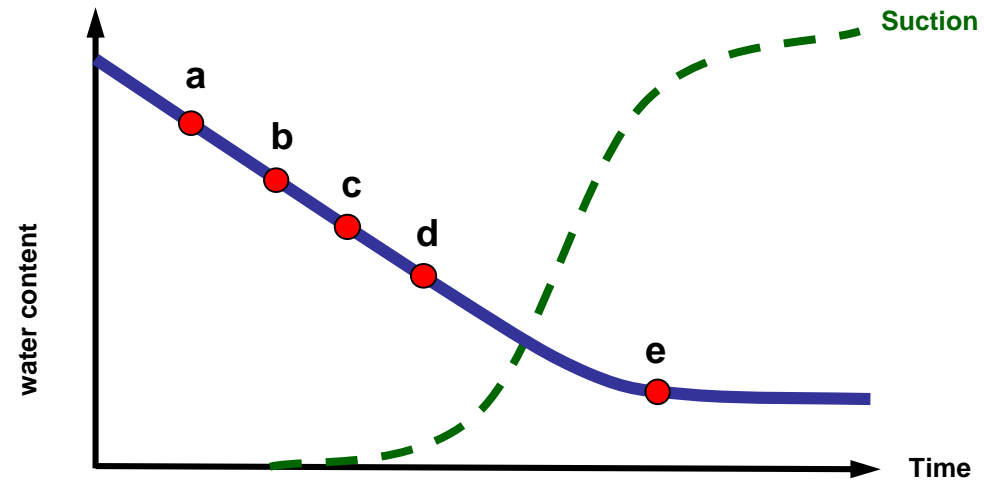
**Permeability**

**Mixed fluids**



BBC News In pictures Visions of Science.jpg

# Evolution

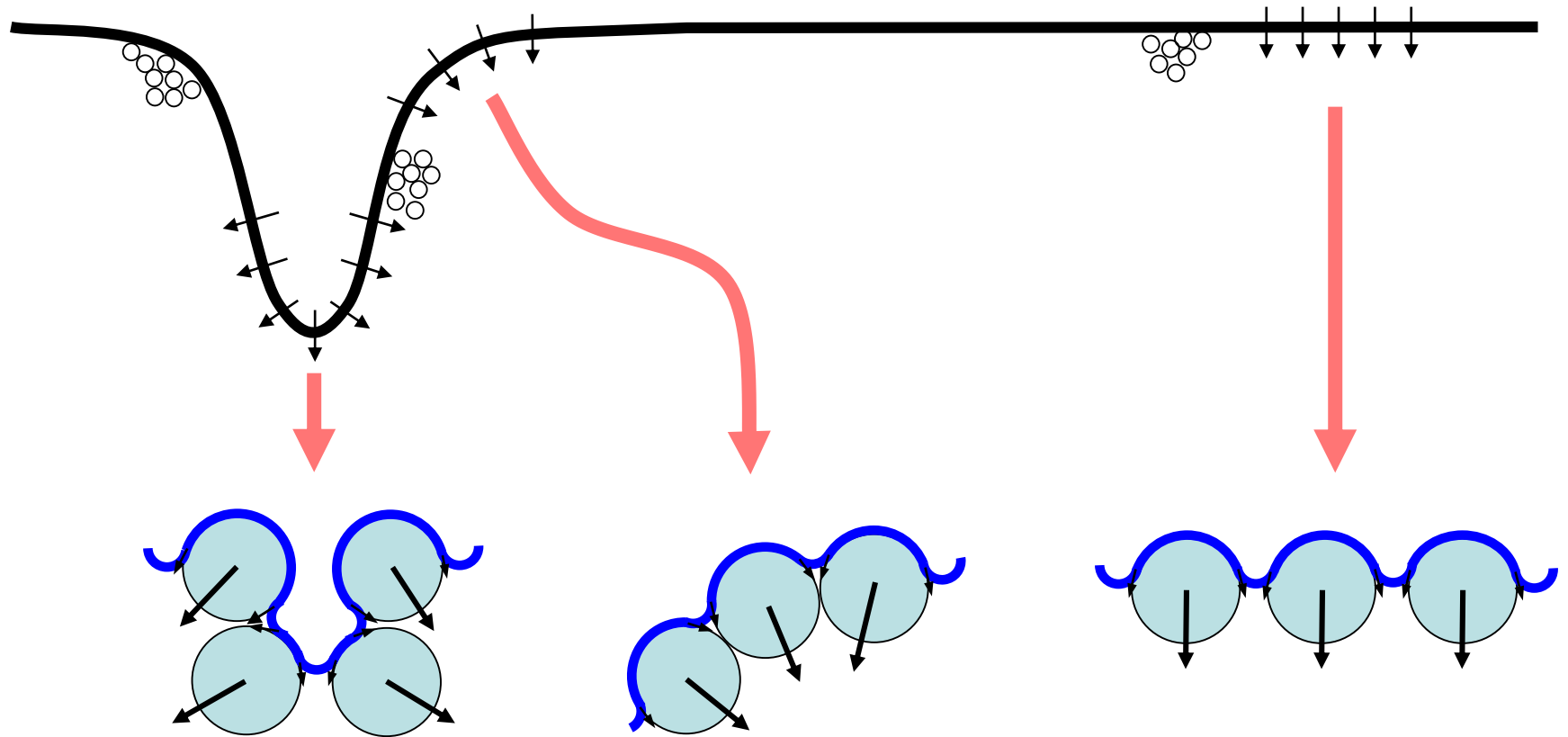




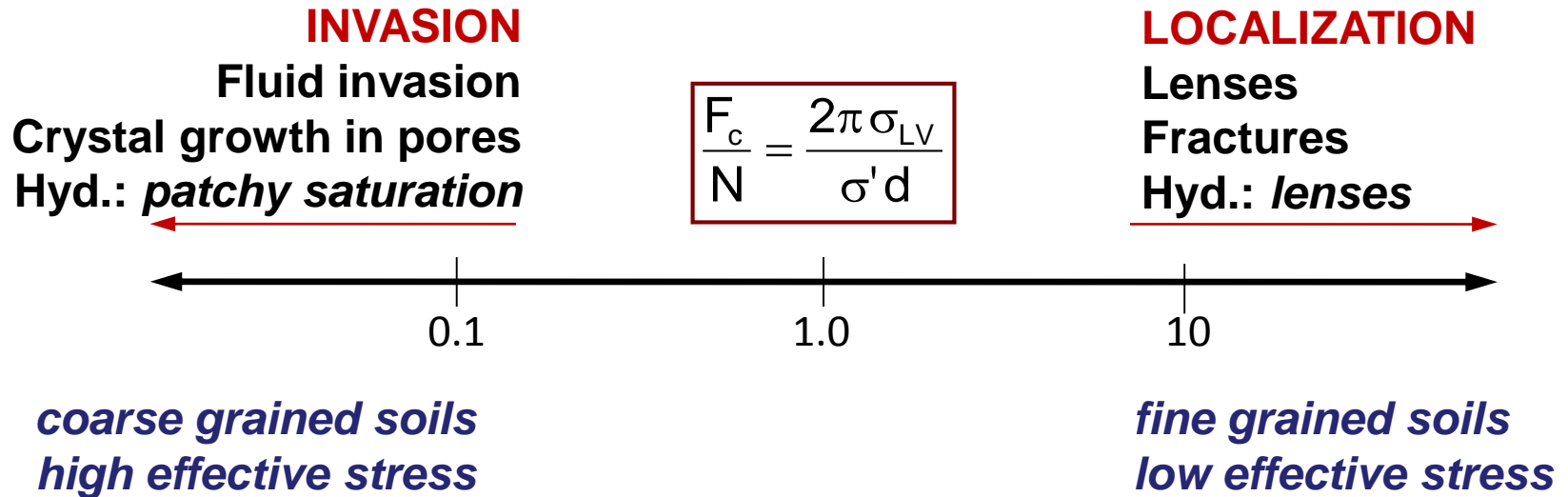
# Forcing Gas Into Sediment



# Gas-Driven Fracture



# Invasion vs. Localization



**Formation**

**Size → Forces**

**Shape**

**Soil Classification**

**Diagenesis**

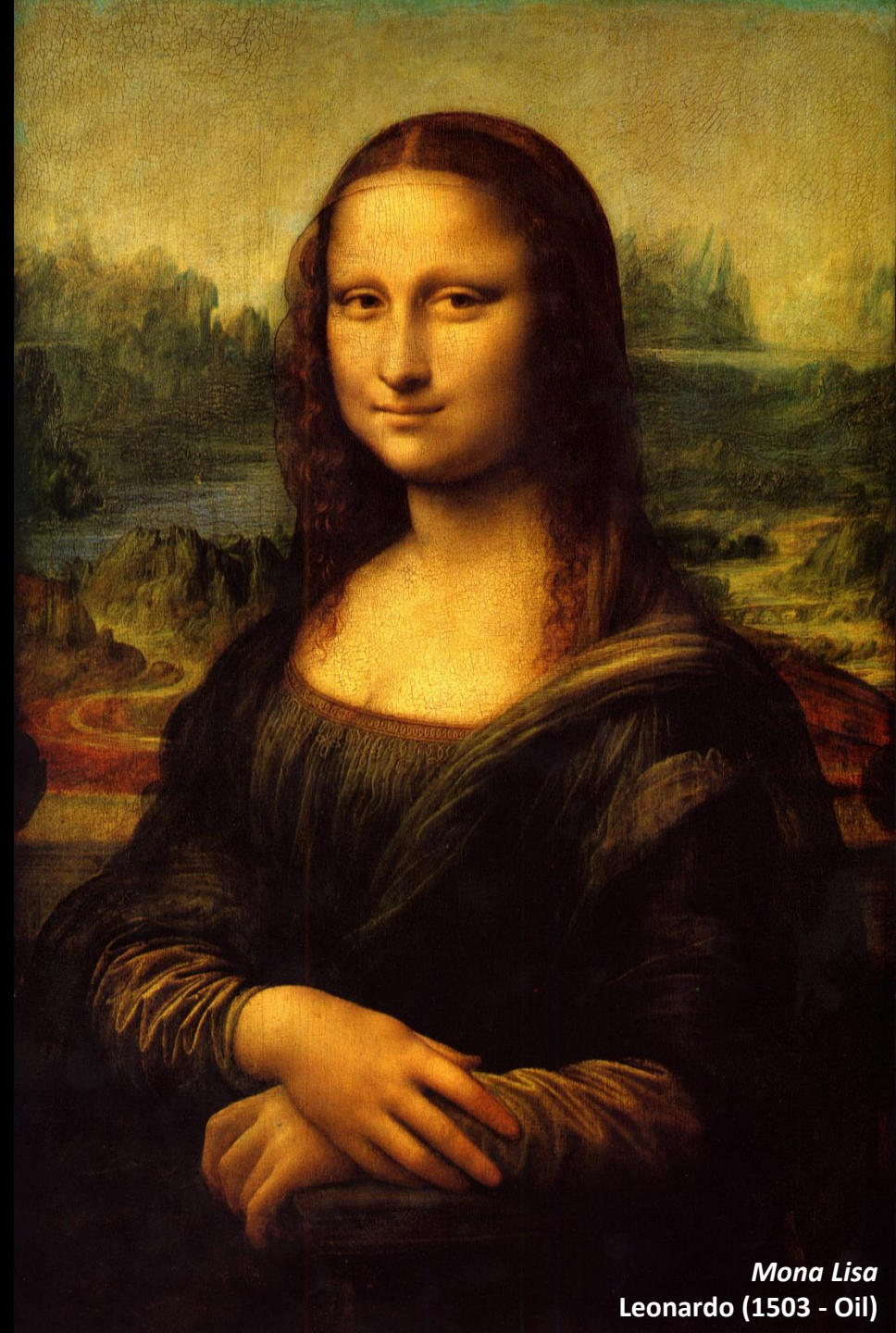
**Shear strength**

**Stiffness**

**Pores**

**Permeability**

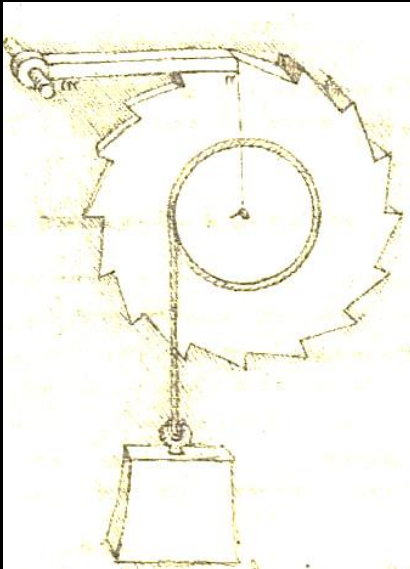
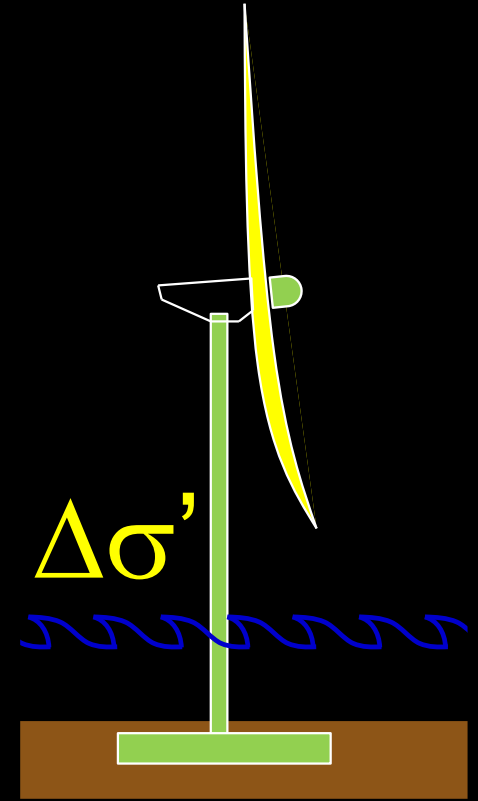
**Mixed fluids**



*Mona Lisa*  
Leonardo (1503 - Oil)

# Repetitive Loads

Leonardo (1452- 1519)



# Instability → Localization

**Strain localization**  
shear band  
compaction band

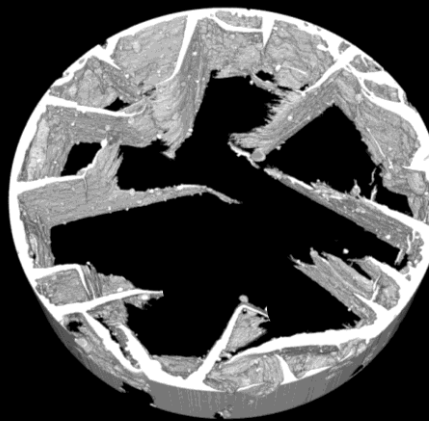
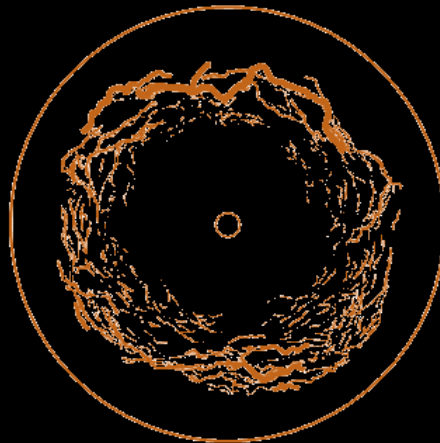
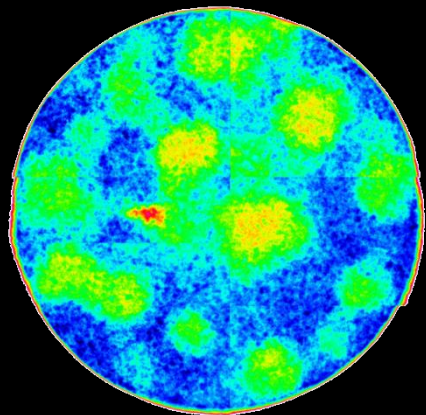
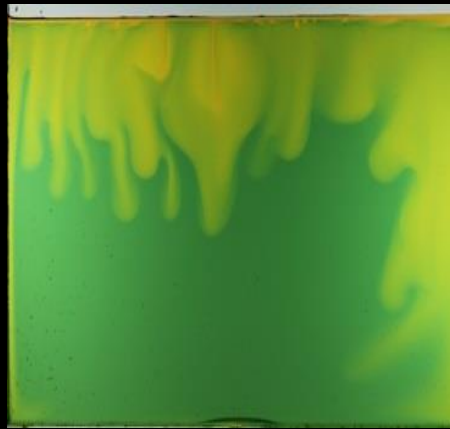
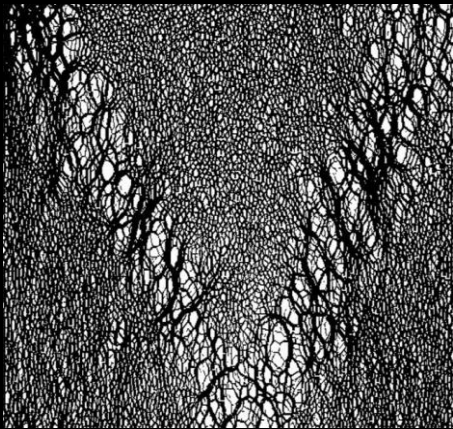
**Grains migration**  
clogging  
piping erosion  
sand production

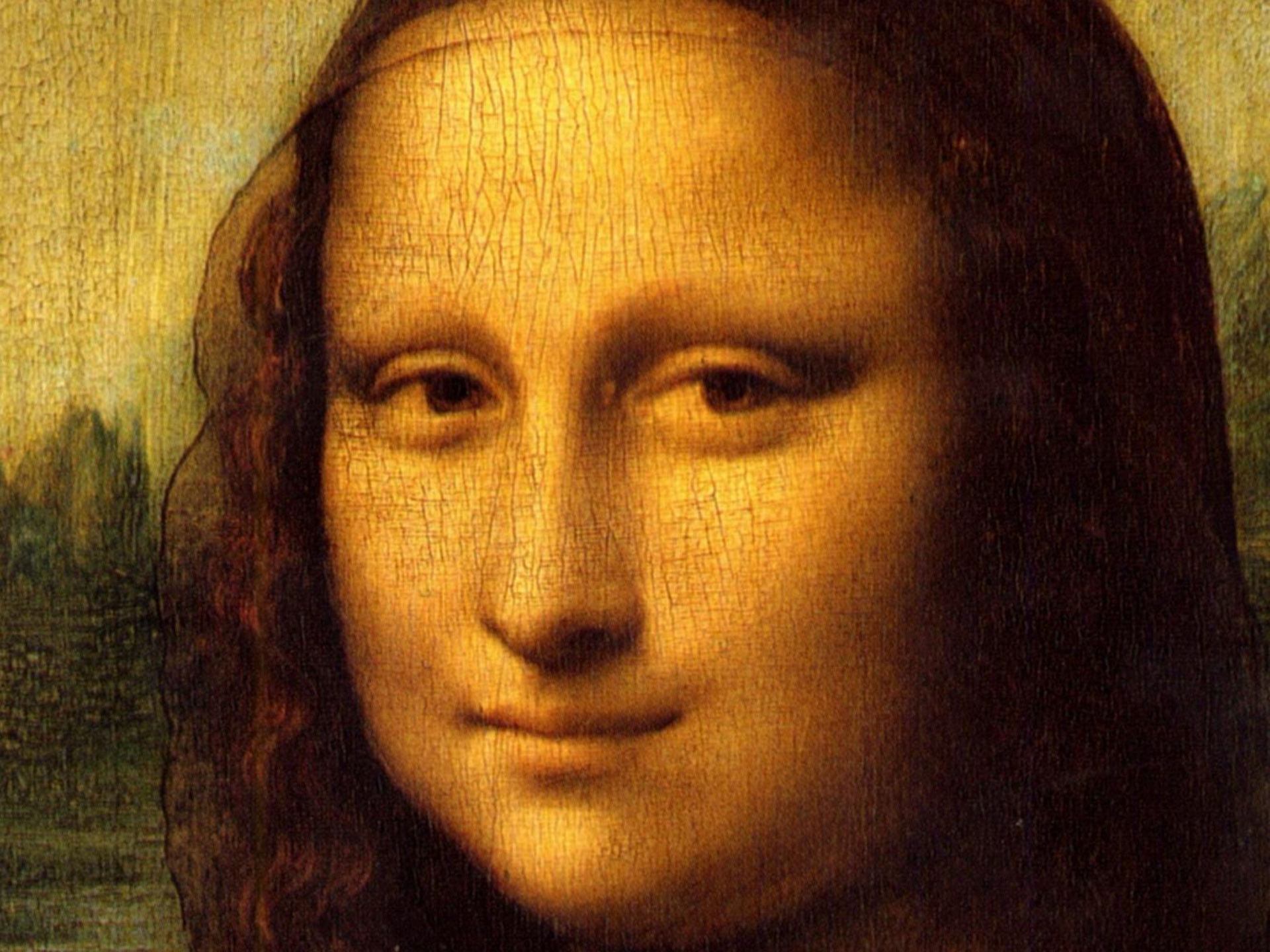
**Fluids**  
viscous fingering  
density tears

**Fluid-driven**  
desiccation cracks  
gas or oil-driven  
hydraulic fracture

**Lenses**  
ice & hydrate

**Dissolution**  
shear in contraction  
piping & wormholes







**Thank you**